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FITTING FIREPOWER SCORE MODELS TO THE BATTLE OF KURSK DATA

By

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This thesis applies several Firepower Score attrition algorithms to real data. These algorithms are used in highly aggregated combat models to predict attrition and movement rates. The quality of the available historical data for validation of attrition models is poor. Most accessible battle data contain only starting sizes and casualties, sometimes only for one side. A detailed database of the Battle of Kursk of World War II, the largest tank battle in history, has recently been developed by Dupuy Institute (TDI). The data is two-sided, time phased (daily), highly detailed, and covers 15 days of the campaign. According to combat engagement intensity, three different data sets are extracted from the Battle of Kursk data. RAND's Situational Force Scoring, Dupuy's QJM and the ATLAS ground attrition algorithms are applied to these data sets. Fitted versus actual personnel and weapon losses are analyzed for the different approaches and data sets. None of the models fits better in all cases. In all of the models and for both sides, the Fighting Combat Unit Data set gives the best fit. All the models tend to overestimates battle casualties, particularly for the Germans.

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FITTING FIREPOWER SCORE MODELS TO THE BATTLE OF KURSK DATA

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First Lieutenant, Turkish Army
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LIST OF SYMBOLS, ACRONYMS AND ABBREVIATIONS

- FPI: Fire Power Index .
- FR: Force Ratio
- FEBA: Forward Edge of the Battle Area
- FLOT: Forward Line of Troops
- ACUD: All Combat Unit Data
- CCUD: Contact Combat Unit Data
- FCUD: Fighting Combat Unit Data
- QJM: Quantified Judgement Models
- SFS: Situational Force Scoring
- TNDM: Tactical Numerical Deterministic Model
- TDI: The Dupuy Institute
- CAA: US Army Concepts Analysis Agency
- ARCAS: The Ardennes Campaign Simulation Study
- KDB: Kursk Data Base
- KOSAVE: The Kursk Operation Simulation and Validation Exercise
- OH: On hand
- HQ: Headquarters
- KIA: Killed In Action
- WIA: Wounded In Action
- CMIA: Captued/Missing In Action
- DNBI: Disease/Nonbattle Injuries
- SP: Self Propelled
- ARTY: Artillery
- APC: Armored Personnel Carrier
- RKTL: Rocket Launcher
- ATH: Heavy Antitank Weapon
- LTH: Light Antitank Weapon
- MTR: Mortar

- Flame/MG: Flame-throwers and heavy machineguns.
- AA Antiaircraft Weapons
- WEI/WUV: Weapon Effectiveness Index, Weighted Unit Value
- KV: Killer-Victim

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EXECUTIVE SUMMARY

"War is a matter of vital importance to the state; the province of life or death; the road to survival or ruin. It is mandatory that it be thoroughly studied." Sun Tzu, one of the most famous military thinkers in history, described war and pointed out the importance of the studies of war over 1500 years ago in his book *The Art of War* [Ref. 1].

Since the dawn of history scientists, researchers, and the military have tried to develop fundamental laws or theories that explain the interactions of military forces in combat and the outcomes of battles. Combat models are widely used in battle planning, wartime operations, force sizing, human resource planning, logistics planning, national policy analysis, and the decision process for the acquisition of weapon systems [Ref. 4:p. 6-7].

Combat models are categorized into two groups according to the level of representation of the combatants. The first group is high-resolution combat models in which each combat vehicle or soldier is explicitly represented as an entity. The second group is low-resolution (aggregated) combat models, which represents battalion and higher units as an entity. The issue of aggregation has been addressed by many authors over the years. Many of these studies on attrition methodology include the subjects of Lanchester's Law, data sources, such as historical vs. engineering vs. empirical, and a combination of the two which are scoring systems such as Firepower Scores. The primary focus of this thesis is the validity of the Firepower Score approach to attrition in low-resolution (aggregated) combat models.

Attrition is a reduction in the number of personnel, weapons, and equipment in a military unit, organization, or force [Ref. 3:p. 1]. Combat attrition is one of the most important aspects of combat modeling. Combat attrition is the only combat process for which well-developed mathematical theories exist [Ref. 6:p. 4-1]. Even though attrition is the most studied combat process, there is no agreement on the best way to model it. The main reason is the lack of real data, which can be used to validate combat attrition models.

Aggregated attrition process models can be categorized into two basic types that correspond to the two basic entity aggregation patterns--homogeneous and heterogeneous [Ref. 6:p. 4-2]. The basic idea of homogenous force ratio attrition models is to aggregate all the individual combatants in a unit into a scalar measure of the unit's combat power [Ref. 6:p. 4-3]. The Firepower score approach is used in, aggregated, large-scale, combat simulations as the primary descriptor of what a combat unit is worth [Ref. 6:p. 2-5]. The ratio of attacker to defender combat power is used to determine the casualties for both sides.

The quality of the available historical data for validation of attrition models is very poor. The most accessible battle data contain only starting sizes and casualties and sometimes only for one side [Ref. 5:p. 470]. Recently, more data has become available. Improved database management and computing power have helped in gathering such data.

Detailed databases on the Battle of Kursk, the largest tank battle in history, and the Ardennes Campaign of World War II, have recently been developed. Both data sets

are two-sided, time-phased (daily) and detailed. Hartley and Helmbod pointed out that unless we are able to procure time-phased detailed data, we will not be able to validate any attrition model [Ref. 5:p. 89]. In this thesis, we focus on time-phased, highly detailed, two-sided data.

Most of the past empirical validation studies have focused on the Lanchester Equations, which were introduced by the English engineer, Frederick William Lanchester. These studies include the works of Bracken [Ref. 8] and Fricker [Ref. 9] on the Ardennes Campaign of World War II, Clemens [Ref. 10] and Turkes [Ref. 11] on the Battle of Kursk of World War II, and Hartley and Helmbold [Ref. 12] on the Inchon-Seoul Campaign of the Korean War. These works are among the few quantitative studies that use daily force size data for real battles.

Besides the Lanchester equations, another approach for combat attrition is models that use force ratio in their structure. This thesis focuses on aggregate attrition methodologies that use combat power ratio to compute the casualties of the forces. Unlike the Lanchester equations, there is no study in the literature that used firepower score attrition models on real data in which force sizes are available day by day for both sides.

One of the interesting aspects of the Battle of Kursk is the engagement percentages of the forces. The Germans had a considerably larger portion of their forces in contact. This suggests that the German force may have been subjected to more fatigue than the Soviets. With an average of 97 percent of its heavy mechanized force on the front lines, the Germans, unlike the Soviets, had no reserves to use [Ref. 14:p. 9-4]. Depending on combat engagement intensity, this study extracts three different data sets

from the data on the Battle of Kursk in the KOSAVE report.

The first data set, all combat units data (ACUD), includes all combat units: contact, out of contact, active, and inactive, including HQ above division level. The second data set, contact combat units data (CCUD), includes only combat units those are in contact with the enemy: units in contact fighting and not fighting, HQ above division level is excluded [Ref. 14:p. 5-9]. The third data set, fighting combat units data (FCUD), includes only combat units that are in contact and fighting with the enemy: HQ above division level are excluded [Ref. 14:p. 5-9].

In this research, three Firepower score models are applied to the three data sets that are extracted from the data on the Battle of Kursk. The first model is the ATLAS ground attrition model, which is used in the TACWAR simulation [Ref. 24]. The second model is RAND's SFS model, which was proposed in 1991 and is used in the JICM simulation [Ref. 25]. The last model is the simplified QJM model, developed by Trevor Dupuy. Instead of focusing only on one model and investigating it in detail, the applicability of the three primary firepower score models to real data is investigated. In addition, some insight is given about the attrition processes and other factors used in aggregated combat models.

The key findings from this research include:

- Of all the models looked at, when combat power losses are considered, the ATLAS model with the air sortie data fits best.
- Generally, the models overestimate the attacker's casualties during the battle.
- Overall, all of the models fit better for the Germans than the Soviets. In his study [Ref. 11], Turkes also found that his models fit better for the Germans.

- In all of the models and for both sides, the FCUD data set gives the best fit.
- One of the difficulties with aggregated combat attrition models that use force ratio is the need to determine the attacking side. It is always not very easy to determine the attacking side.
- Prior to a battle, it is difficult to determine factors such as intensity and nationality factors.
- One of the problems with traditional force ratio models is that the loss rates in each weapons category are the same as the combat power casualty rate. For instance, if the combat casualty rate is 4 percent, then each weapons category will take 4 percent losses. However, this does not match either the historical facts or the results from the higher-resolution combat models. In this thesis, the linear regression analysis is used to determine how to allocate the combat power casualty rates to the different weapon groups.
- Due to the general overestimation of the German casualties and the underestimation of the Soviet casualties, anything that improves the force ratio with respect to the Germans improves the quality of the fits. Anything that added to German effectiveness or cut Soviet effectiveness could improve the quality of the fits.
- Using different firepower scores, like Bracken's weights, does not give a better fit except for the Soviet values in the CCUD data set, which is slightly better. Much more work is needed to find the best firepower scores, such as optimization of the score values, and sensitivity analysis.
- The FLOT movement rate is only computed in the SFS model. The ATLAS model has also look up tables to compute the movement rates according to the force ratio, terrain, and combat postures. However, in the tables, the force ratio threshold is higher than the ones computed for this battle. The force ratio of the Germans in the ATLAS model is very low. As a result, it was not possible to compute the FLOT movement rates in the ATLAS model.
- This analysis is based on observational census data of the Battle of Kursk of World War II, and may not generalize, since it is not a random sample of a larger population. The outcome of a battle cannot be precisely determined with the use of combat models. They might provide insights into future battles between adversaries. Besides being used to gain insight into the battles, which occurred in the past, they should help in making better decisions by enabling the decision-maker to compare the different alternatives by using various combat model techniques [Ref. 11:p 145].

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I. INTRODUCTION

"War is a matter of vital importance to the state; the province of life or death; the road to survival or ruin. It is mandatory that it be thoroughly studied." Sun Tzu, one of the most famous military thinkers in history, described war and pointed out the importance of the studies of war over 1500 years ago in his book *The Art of War* [Ref. 1]. Clausewitz defined war in his book, *On War* [Ref. 2], as "War is thus an act of force to compel our enemy to do our will." Throughout history, war has been a topic of analysis for scientists and researchers.

A. COMBAT MODELING

An understanding of combat phenomena is facilitated by using a hierarchy of combat to describe combat events and aggregate them for analysis [Ref. 3:p. 153]. A commonly accepted hierarchy of combat is as follows: *War*, already defined, is at the top of the hierarchy. A *campaign* is a phase of war involving a series of battles related in time and space with the aim of achieving a single, specific objective. A *battle* is combat between major forces, each having opposing assigned or perceived operational business. An *engagement* is combat between two forces from battalion to division level. An *action* is combat between squad or battalion level. A *duel* is combat between two individuals.

Since the dawn of history scientists, researchers, and the military have tried to develop fundamental laws or theories that explain the interactions of military forces in combat and the outcomes of battles. Combat models are widely used in battle planning,

wartime operations, force sizing, human resource planning, logistics planning, national policy analysis, and the decision process for the acquisition of weapon systems [Ref. 4:p. 6-7].

In Operation Desert Storm an existing theater-level simulation, Concept Evaluation Model (CEM), was used by analysts at the US Army Concepts Analysis Agency (CAA) [Ref. 5:p. 549] to simulate the Desert Storm scenario for assisting in war-planning and war-fighting efforts. The actual efforts of their studies became clear when a letter from LTG Reimer to the Director of CAA, Mr. E.B. Vandiver, arrived in February 1991. The letter, in part, stated, "The analytical support you provided for Operation Desert Shield and Desert Storm has been absolutely outstanding. [It was] used by the Army Staff, the Joint Staff and our Army in Southwest Asia to prepare for war. The Army leadership used it for discussion and briefings with key military and civilian leaders, including the National Command Authority [Ref. 5:p. 559]."

Combat models are categorized into two groups according to the level of representation of the combatants. The first group is high-resolution combat models in which each combat vehicle or soldier is explicitly represented as an entity. The second group is low-resolution (aggregated) combat models, which represents battalion and higher units as an entity. The issue of aggregation has been addressed by many authors over the years. Many of these studies on attrition methodology include the subjects of Lanchester's Law, data sources, such as historical vs. engineering vs. empirical, and a combination of the two which are scoring systems such as Firepower Scores. The primary

focus of this thesis is the validity of the Firepower Score approach to attrition in low-resolution (aggregated) combat models.

B. ATTRITION

Attrition is a reduction in the number of personnel, weapons, and equipment in a military unit, organization, or force [Ref. 3:p. 1]. Combat attrition is one of the most important aspects of combat modeling. Combat attrition is the only combat process for which well-developed mathematical theories exist [Ref. 6:p. 4-1]. Even though attrition is the most studied combat process, there is no agreement on the best way to model it. The main reason is the lack of real data, which can be used to validate combat attrition models. It is useful to be able to predict combat attrition accurately in order to provide estimates of requirements for the planning process for medical, logistics and personnel training [Ref. 3:p. 2].

Aggregated attrition process models can be categorized into two basic types that correspond to the two basic entity aggregation patterns--homogeneous and heterogeneous [Ref. 6:p. 4-2]. A heterogeneous aggregated attrition model assesses the amount of attrition caused by a weapon system class against each enemy weapon system class. Thus, the interactions between different weapon groups (i.e., who kills who) are implemented in this type of attrition processes. In a homogenous aggregated attrition process, all of the weapon groups are aggregated with their weights into the combat power of a single unit. Most homogeneous attrition models determine the amount of combat power attrition by

computing attacker to defender force ratios [Ref. 6:p 4-2]. The interactions between different weapon groups are not considered in homogeneous attrition models.

C. FIREPOWER SCORE APPROACH

The basic idea of homogenous force ratio attrition models is to aggregate all the individual combatants in a unit into a scalar measure of the unit's combat power [Ref. 6:p. 4-3]. The Firepower score approach is used in, aggregated, large-scale, combat simulations as the primary descriptor of what a combat unit is worth. [Ref. 6:p. 2-5]. The ratio of attacker to defender combat power is used to determine the casualties for both sides.

In the Firepower score approach, the combat power of a unit is computed by summing the combat power value for each weapon system in the unit. In Parry's notes [Ref. 6:p. 4-5] the combat power computation is given in a simple equation as follows: Suppose that there are n different types of weapon system in a combat unit and that:

Xi: the number of weapons of type i in the unit [i=1,2,3...n]

Si: the firepower score value representing the combat power for each type i weapon. Then, the firepower index of the aggregated unit is

$$FPI = \sum_{i-1}^{n} X_i * S_i \qquad (1.1)$$

Finally, the force ratio is determined as:

$$FR = FPI\ (A) / FPI\ (D) \qquad (1.2)$$

Where:

FPI(A): the firepower index of the attacking forces

FPI(D): the firepower index of the defender.

The force ratio gives a measure of relative combat power in the battle. The force ratio in many aggregated combat models, such as TACWAR, is used to compute casualties for both sides in a battle and to determine the FEBA (forward edge of the battle area) or FLOT (forward line of troops) movement rates.

The method of determining the firepower scores is a very difficult problem. There are several methods of computing firepower score values, such as military judgement and experience (RAND's ground force scoring system [Ref. 7]), historical combat performance derived from WWII and the Korean War, and results from high resolution simulations (i.g., Anti-Potential-Potential Method) [Ref. 6:p. 2-6].

There is no published validation study in the literature using firepower score approaches on real data in which force sizes are available day by day for both sides. This thesis describes how different firepower score approaches fit to the data on the Battle of Kursk. This data is time-phased, two-sided and very detailed. This study will help analysts make better decisions and perhaps provide a better understanding of war by adding to an understanding how combat models fit to real data. The next section presents the outline of the thesis.

D. THESIS OUTLINE

This thesis consists of five chapters. This first chapter introduces the general concept of combat modeling and firepower score approaches used in the attrition process of aggregated combat models. The second chapter gives a brief history of the Battle of Kursk of World War II and analyzes the battle's data. Three different data sets are extracted from the data on the Battle of Kursk according to the combat engagement intensity. These data sets are all combat units data (ACUD), contact combat units data (CCUD), and fighting combat units data (FCUD).

In the fourth chapter, three force ratio attrition models that use the firepower score approach are applied to the three data sets described above. Chapter five presents the final conclusions and recommendations based on the results and also recommends future areas of study in combat modeling.

II. PREVIOUS VALIDATION STUDIES ON COMBAT MODELING

A. PREVIOUS STUDIES WITH LANCHESTER EQUATIONS

The quality of the available historical data for validation of attrition models is very poor. The most accessible battle data contain only starting sizes and casualties and sometimes only for one side [Ref. 5:p. 470]. Recently, more data has become available. Improved database management and computing power have helped in gathering such data.

Detailed databases on the Battle of Kursk, the largest tank battle in history, and the Ardennes Campaign of World War II, have recently been developed. Both data sets are two-sided, time-phased (daily) and detailed. Hartley and Helmbod pointed out that unless we are able to procure time-phased detailed data, we will not be able to validate any attrition model [Ref. 5:p. 89]. In this thesis, we focus on time-phased, highly detailed, two-sided data.

Most of the past empirical validation studies have focused on the Lanchester Equations, which were introduced by the English engineer, Frederick William Lanchester. These studies include the works of Bracken [Ref. 8] and Fricker [Ref. 9] on the Ardennes Campaign of World War II, Clemens [Ref. 10] and Turkes [Ref. 11] on the Battle of Kursk of World War II, and Hartley and Helmbold [Ref. 12] on the Inchon-Seoul

Campaign of the Korean War. These works are among the few quantitative studies t use daily force size data for real battles.

In his study [Ref. 8], Bracken found that the Lanchester linear model best fits the Ardennes campaign data. Fricker [Ref.7] revisited Bracken's modeling of the Ardennes campaign. In contrast to Bracken, Fricker found that the Lanchester linear and square laws do not fit the data. He concludes that a new form of the Lanchester equations, with a physical interpretation closest to Lanchester's logarithmic law, applies best.

Clemens [Ref. 10] applied the Lanchester Equations to the data on the Battle of Kursk. Clemens used two estimation techniques: linear regression and Newton-Raphson iteration. He concludes that neither the Lanchester linear nor the Lanchester square model fit the data. The Lanchester logarithmic model fits better than the Lanchester linear and square models.

Hartley and Helmbold's study [Ref. 12] focused on validating the homogeneous Lanchester square law by using Inchon-Seoul Campaign data. Hartley and Helmbold use three analysis techniques to examine the data: linear regression, the Akaike Information Criterion (AIC) and Bozdogan's consistent AIC (CAIC). They find that the data do not fit a constant coefficient Lanchester square law. They conclude that, by dividing the campaign into three distinct battles that each battle's data can be fit to a constant coefficient Lanchester square law, using separate coefficients for each battle.

In his study Turkes [Ref. 11] applies a total of 39 diverse models to the data on the Battle of Kursk using different approaches. These approaches include applying the

methodologies of previous studies, using robust LTS (least trimmed squares) regression, including the air sortie data of the battle, considering the battle in separate phases, fitting basic Lanchester equations and using different weights [Ref 9]. He concludes that:

- None of the original Lanchester equations applies very well to the data on the Battle of Kursk. The best fits are implausible.
- The parameters derived from Bracken and Fricker's Ardennes studies do not apply to the data on the Battle of Kursk. This implies that there are no unique parameters that apply to all battles.
- The Robust LTS regression method is the best analytic technique for estimation of parameters.

B. PREVIOUS STUDIES WITH FIREPOWER SCORE APPROACHES

Besides the Lanchester equations, another approach for combat attrition is models that use force ratio in their structure. This thesis focuses on aggregate attrition methodologies that use combat power ratio to compute the casualties of the force. Unlike the Lanchester equations, there is no study in the literature that used firepower score attrition models on real data in which force sizes are available day by day for both sides.

In this research, the ATLAS ground attrition equations [Ref. 13], RAND's Situational Force Scoring (SFS) [Ref. 7] and Dupuy's Quantified Judgement Models (QJM) [Ref. 3] are applied to three data sets that are extracted from the data on the Battle of Kursk.

The ATLAS theater level simulation uses a straightforward force ratio method. The simplicity of its structure is one of the main attractions of the ATLAS model. TACWAR is one of the simulations that use the ATLAS equations. In the combat attrition process of the ATLAS model, the casualty rates are determined by using simple

equations for the attacker and the defender. The original casualty rates used in the ATLAS model were derived from data on 37 division level engagements in World War II and Korea [Ref. 6:p. 4-9]. Since the specific engagements are not documented, it is unknown as to whether the division-level data includes the battle of Kursk—though it is believed not to. If it is included, then the comparisons are not strictly independent. However, since Kursk would be the only one of a large number of engagements (37) the dependence will be very small. There is no published study on the validation of ATLAS equations for real combat data.

The second method used in this study is RAND's SFS methodology, which was proposed by Patrick Allen in 1991 [Ref. 7]. The SFS methodology has been developed to better account for situation-dependent combined arm's effects in aggregate combat models [Ref. 7:p. 1]. In the SFS method, the value of a weapon system is varied as a function of the combat situation, defined by type of terrain and type of battle, and as a function of shortages in the weapon mix in a given combat situation. The ratio of attacking combat power to the defending combat power is defined as the "situationally adjusted" or "modified" force ratio (MFR). The SFS's equations use this modified force ratio to compute the casualty rates and FEBA (FLOT) movement rates.

The basis for the equations used in the SFS was documented in an unpublished work by Paul Davis and Patrick Allen in the mid-1980s [Ref. 7:p. 41]. There has been no effort to date to calibrate these equations or their parameters. Also, this method is not applied to any real two-sided, daily combat data.

The last method used in this research is Dupuy's QJM model. In his book, *Attrition* [Ref. 3], Dupuy presents simple equations to predict the personnel and material losses of a military force. These equations are incorporated in the Quantified Judgment Model (QJM) and the Tactical Numerical Deterministic Model (TNDM), both of which were developed by Dupuy [Ref. 3:p. 104].

Dupuy applied his methodology retrospectively to a number of historical battles from 1805 to 1973, with quite good results [Ref. 3.p. 113]. All the data used in his examples contain only the starting and ending force strength and casualties mostly for personnel and armor assets. At the end, he presents a scale (such as fair, excellent, phenomenal) to show his subjective assessments of the relative quality of the forecasts or estimates. For personnel estimates, the results are as follows [Ref. 3:p. 124]:

- Total data sets: 25
- Fair: 3
- Good: 6
- Excellent: 4
- Phenomenal: 12

For armor estimates the results are as follows [Ref. 3:p. 124]:

- Total data sets: 8
- Good: 3
- Excellent: 1
- Phenomenal: 4

Overall, the average deviation for personnel estimates is 9.6%. The average deviation for armor estimates is 9.0% [Ref. 3:p. 124].

C. STUDY METHODOLOGY

This thesis applies the firepower score attrition models to the data on the Battle of Kursk. The two main areas of interest are the quality of the fits and the insights provided by the models. Different models will be compared and contrasted. The methodology used in this thesis research consists of the following steps:

- Arranging and setting up the data on hand to be useful for analysis.
- Conducting a through analysis and interpretation of the data.
- Extracting three data sets from the original data according to the combat engagement status of the units.
- Applying the ATLAS ground attrition equations to all data sets.
- Evaluate the fit of the ATLAS equations apply to the three data sets.
- Applying the SFS methodology to the three data sets.
- Evaluate the fit of the SFS methodology to the data on the Battle of Kursk.
- Evaluate the fit of the FLOT movement rates in the SFS methodology to the Battle of Kursk.
- Applying the QJM models to the Battle of Kursk.
- Evaluate the fit of the QJM models apply to three data sets.
- Distribute the combat power losses into different types of weapons.
- Including the air sorties to the data.
- Comparing and contrasting different methodologies.
- Analyzing the results and conclusions of all the models.

III. HISTORY AND DATA ON THE BATTLE OF KURSK

A. A SHORT HISTORY ON THE BATTLE OF KURSK

In the spring of 1943, the Russo-German front was dominated by a salient located to the north of Kharkov, to the south of Orel, and centered in the city of Kursk. The Kursk salient had a frontage of 250 miles and 70 miles across its base [Ref. 14:p. 2-2].

In order to regain the initiative in Russia after the reverses in the winter campaign of 1942-1943, and to strengthen the front line, Hitler decided to launch an offensive operation known by the code name "Citadel (Zitadelle)" [Ref. 15:p 152] Through this attack, Hitler wanted to considerably strengthen the front in the Belgorod-Orel area. The Donets Basin was of great economic importance. Since the front line passed directly along the eastern edge of the basin, Hitler considered it too insecure and vulnerable to enemy attack [Ref. 15:p 153]. The German plan was a two-front attack on the Kursk salient in a classic pincer operation.

Initially the attack was to be launched on 4 May 1943, however, the attack was postponed until 5 July 1943. Postponement of the attack from May to July 1943 subsequently proved to be a great disadvantage to the Germans. Although Hitler argued that the delay was necessary in view of an anticipated Allied attack on the Italian coast, it was clear that the longer the Germans delayed, the more probable it was that the Soviets would develop defenses to thwart the attack.

Operation Citadel was launched on July 5, 1943, see Figure 3.1. Using a massive armor attack, General Model's 9th Army attacked the northern front of the salient, while General Hoth's 4th Panzer Army attacked from the southern front [Ref. 14:p. 2-2]. German forces encountered heavy losses as they fell upon the prepared Soviet positions, which contributed, to Germany's defeat in this campaign.

After an initial gain of a few miles in the first two days of the battle, the 4th Panzer Army surged forward on 7 July, creating great damage and alarm among Soviet positions [Ref. 14:p. 2-3]. Despite these heavy losses in men and armor, Soviet reinforcements were sufficient to restrict the German gain to 25 miles by 12 July.

On 12 July, a German breakthrough attempt resulted in a major close quarters tank battle near the town of Prokhorovka. This day was a turning point in the battle and described in the *The Battle of Kursk* [Ref. 16] as " Immense in scope, ferocious in nature, and epic in consequences, the Battle of Kursk witnessed (at Prokhorovka) one of the largest tank engagements in world history and led to staggering losses." Unable to gain a decisive victory, the Germans drew back into generally defensive postures after this battle.

Hitler canceled Operation Citadel on 13 July, and later German attacks were limited in scope. The Soviets began counterattacks on the southern front on 12 July but shifted primarily defense postures by 14 July. The Soviet counteroffensive resumed on 18 July and they regained all of the ground lost in the theatre by July 23, 1943. [Ref. 14:p. 2-3]

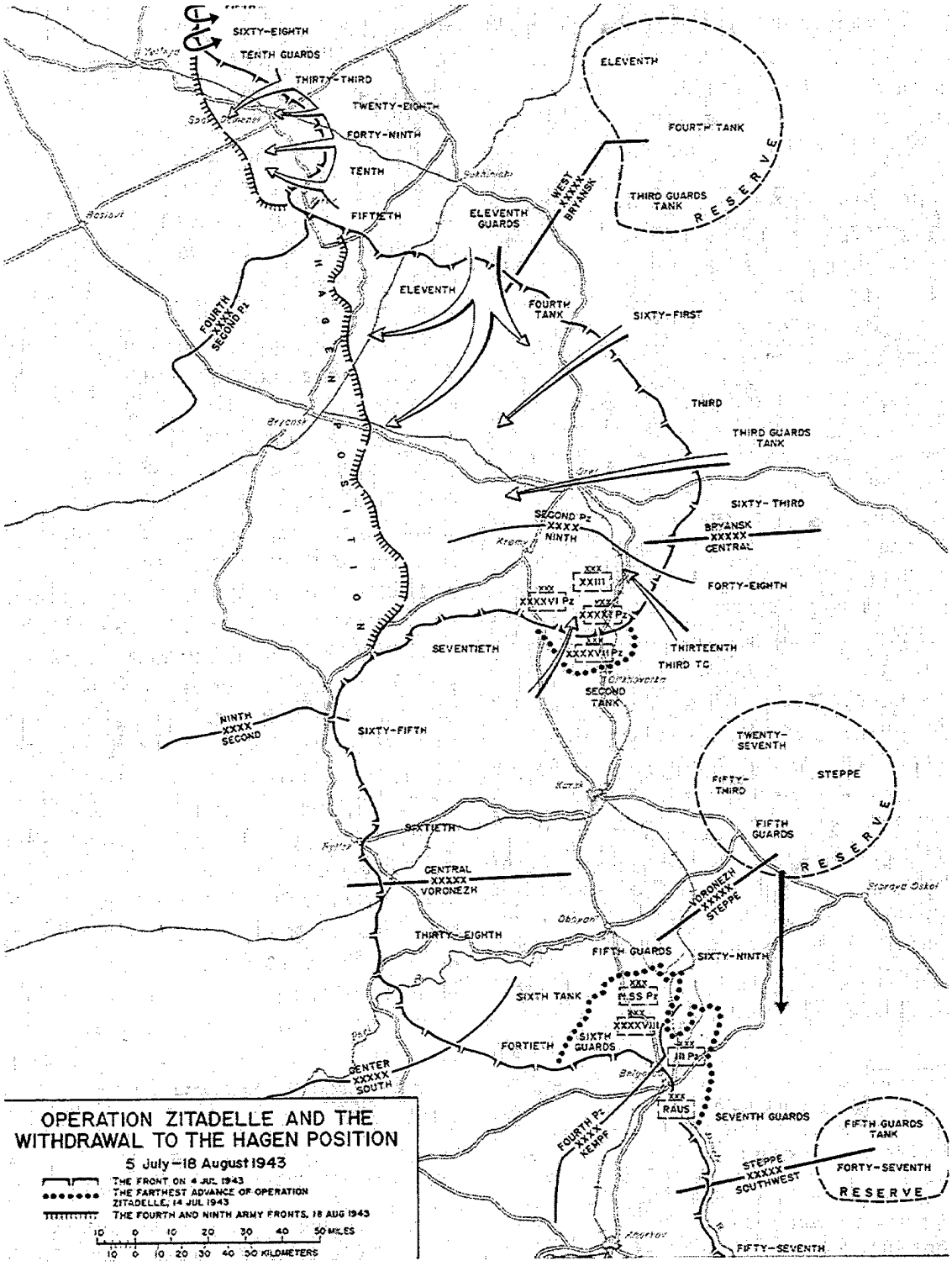

Figure 3.1. Operation Zitadelle [From Ref. 17:p. 134].

B. DATA ON THE BATTLE OF KURSK

This section presents the procedure for the extraction of data sets used in this study. This process was the most difficult and time-consuming process of the study.

1. Creation and Scope of the Kursk Database

In order to improve combat model credibility, the Ardennes Campaign Simulation (ARCAS) Study [Ref. 18] compared a computerized combat model representation of the World War II (WWII) 1944-1945 Ardennes Campaign with a database of historical results from that campaign. This comparison was used to assess the accuracy of the simulation model and to develop algorithmic changes [Ref. 14:p. 1-1].

Another comparative historical campaign is necessary to test the ARCAS simulation. The Dupuy Institute (TDI), under contract to the US Army Concepts Analysis Agency (CAA), collected historical data from forces on the southern front of the Battle of Kursk, 5 July 1943 through 18 July 1943, from military archives in Germany and Russia [Ref. 14:p. 1-1]. This data was reformatted as the Kursk DataBase (KDB). The Kursk Operation Simulation and Validation Exercise (KOSAVE) Study, a follow-on effort to the ARCAS Study of 1995, was initiated to compare simulated campaign results with history [Ref. 14:p. 1-1].

The results and products of this simulation are presented in report CAA-SR-98-7 [Ref. 14:p. 1-1]. Additional supplemental data was released with this report on a CD-ROM [Ref. 19]. All of the data used in this research is extracted from this CD-ROM.

The KOSAVE report includes only the southern front of the World War II (WWII) Battle of Kursk, as represented in the KDB historical data [Ref. 14:p. v]. Only

the results and data for combat units in the KDB are included. Non-combat support units are not covered in this report.

2. Limitations and Timeframe of the Kursk Database

In the KOSAVE report, results are not expressed in terms of specific weapon types; instead, weapons are aggregated into categories or classes. Human factors such as leadership, morale, fatigue, caution, and aggressiveness are not quantified. The timeframe for the data is from 4 July 1943 through 18 July 1943.

3. Assumptions for the Kursk Database

There are three primary assumptions made for the KDB database. The database accurately represents the status and structure of forces in the southern front of the actual Battle of Kursk. The personnel casualty and system kill criteria used to categorize KDB casualty and weapon losses are sufficiently consistent with each other to allow meaningful reporting and comparisons between combatants. The use of interpolation techniques for gathering data between inconsistent reports in historical records to create a complete set of daily report records in the KDB is reasonable [Ref. 14:p. 1-3].

C. METHODOLOGY USED FOR THE EXTRACTION OF THE DATA

The data used throughout this study is extracted from the KOSAVE [Ref. 14] report. All of the data used in this study are for combat units represented in the KDB. Support units, such as bridging and logistic units, are excluded in the KOSAVE study.

Twenty-four primary German combat units are represented in the KDB. These 24 primary combat units are composed of 7 headquarters (HQ) units and 17 line units. The

17 line units are further partitioned into 8 infantry divisions (IDs), 5 Panzer divisions (PzDs), and 4 Panzer Grenadier divisions (PzGrDs) [Ref. 14:p. 3-1]. The German HQ units represented are corps and army level.

There are 67 primary Soviet Combat units represented in the KDB. These are 19 Soviet headquarters units and the 48 line units. The 48 line units are further partitioned into 35 rifle divisions, 8 tank corps, 2 mechanized corps, 2 airborne divisions, and 1 detachment [Ref. 14:p. 1-3]. The Soviet HQ unit types are armies, rifle corps and the Voronezh Front.

1. Personnel Data

German and Soviet personnel strengths are used to represent the combat manpower of combat units for both forces in the southern front Kursk Battle. The personnel strengths are presented as "onhand" (OH) which represents the available combat manpower.

Personnel strength losses are killed in action (KIA), wounded in action (WIA), and captured/missing in action (CMIA). Disease and nonbattle injuries (DNBI) are not counted as combat losses. Since DNBI are not caused directly by the enemy and only combat units are taken into account, they are not considered as combat losses in this study. Upon comparing this study's results with the previous studies, this classification should be considered.

2. Weapons Data

The many specific weapon types listed in the KDB were grouped into nine weapon classes in order to achieve compact summarization [Ref. 14:p. 5-1]. We used this weapon categorization in appropriate models, but in some models weapons are categorized differently. The methodology used to demonstrate how data is gathered for modeling purposes is explained in the section concerning that specific model. The OH weapon numbers represents the available number of weapons in that category.

Weapon losses are categorized as damaged and destroyed/abandoned. Damaged weapons are considered as a loss, because a damaged weapon system is considered to be a "temporary" loss and in a non-operational status. A damaged weapon system is treated as only a "temporary loss," but the period of non-operational status can be long. Also, a damaged system will function only with degraded effectiveness and efficiency [Ref. 11:p. 27].

The weapon classes used in the KOSAVE report are: tanks (Tank), armored personnel carriers (APC), artillery (ARTY), rocket launchers (RKTL), heavy antitank weapons (ATH), mortar (MTR), light antitank weapons (ATL), flame-throwers and heavy machineguns (Flame/MG), and antiaircraft weapons (AA). All weapons of each specified KDB weapon type are in only a single weapon class. Most assignments of weapon types to classes are the same as those used in the KDB documentation. A weapon type with dual capability (e.g., antitank and artillery) is assigned to the weapon class representing what was judged to be its primary function in the battle [Ref. 14:p. 5-1].

The KOSAVE study gives detailed information for the composition of weapon classes in Tables 5.1 and 5.2 [Ref. 19:Data98-7, worksheet CHAP5T]. Also, it presents the OH and losses of all specific weapon types for all combat units. The Data98-7 is in a Microsoft Excel file on the CD-ROM [Ref. 19], and the CHAP5T is a Microsoft Excel worksheet in the Data98-7 file. The OH weapon numbers are given on a CD-ROM [Ref. 19:Data98-7, worksheet CHAP5] in nine groups for both sides.

In order to prevent possible confusion on the part of future analysts, the methodology used for gathering the data for the categorization of weapon types will be detailed. Each weapon group type is listed below. Tables 5.1, and 5.2 [Ref. 19:Data98-7, worksheet CHAP5T], are used for this classification.

a. Classification of German Weapon Types

(1) Tanks used in the study:

- PzIII, PzIV, PzV, PzVI
- T-34(Soviet), PzIIIspt

The PzIII Flame and PzIII Observations in the original KDB documentation where defined as PzIIIspt in the KOSAVE report [Ref. 19:Gewpns, worksheet KDBGEWPNTYPES]. Also, according to Table 5-1 [Ref. 19:Data98-7, worksheet CHAP5T], the PzIII Flame is assigned to the Flame/MG weapon category. When the number of each weapon type defined in Table 5-1 was summed from the OH weapon numbers [Ref. 19:Gewpns, worksheet GEOHWPNS], the total number was not the same as the number of the Flame/MG weapon group shown in Figure 5-5 [Ref. 19:Data98-7, worksheet CHAP5]. Further analysis showed that the PzIIIspt should be placed in the tank weapon category [Ref. 20].

(2) Artillery used in the study:

- AC8w 75mm, MHT75mmIG, 75mm lt IG
- 87.6mm How, 105mm How, 150mm How, 152mm How, 155mm How, 210mm How
- 105mm Gun, 150mm Gun
- Wespe, StuH, Grille, Hummel

(3) APC used in the study:

- AC4-6w, AC8w, LHT, MHT
- LHT81mmmtr, LHT37mmAT, MHT37mmAT
- LHTspt, MHTspt, ACspt
- Pz I, Pz II, MHT Flame

There are also some errors in the classification of the APCs in the KOSAVE study. The MHT Flame is in the Flame/MG weapon group, but when it is included in this weapon group, the total numbers do not match as described above in the tank categorization. The second problem is that the LHT81mmmtr is defined in the MTR weapon group. Once again, the total MTR OH number in Figure 5-5 [Ref.12:Data98-7, worksheet CHAP5] do not match the number obtained from the summation of each MTR weapon types defined in Table 5-1 [Ref. 19:Gewpns, worksheet GEOHWPNS]. Both the MHT Flame and LHT81mmmtr weapon types are defined in the APC weapon category in this study.

(4) Rocket Launchers used in this study:

- 150mm Lnch
- 210mm Lnch
- 280mm Lnch

(5) Heavy antitank (ATH) weapons used in this study:

- 75mm AT
- MarderII, MarderIII

- StuG III

(6) Mortars (MTR) used in this study:

- 50mm mtr, 81mm mtr
- MHT81mmmtr, 82mm mtr

As mentioned above in the APC classification, although the LHT81mmmtr is in the MTR weapon group in Table 5-1[Ref. 19:Data98-7, worksheet CHAP5T], it is put into APC weapon group.

(7) Light antitank (ATH) weapons used in this study:

- 45/50mm AT, 28/20mm AT
- 37mm AT, ATR

(8) Flame/MG used in this study:

- Flamethr
- MMG

Although the MHT Flame and Pz III Flame are in the Flame/MG weapon group in Table 5-1[Ref. 19:Data98-7, worksheet CHAP5T], the MHT Flame is put into the APC and the Pz III Flame is put into tank weapon groups.

(9) Antiaircraft (AA) weapons used in this study

- 20mm AA
- 37mm AA, 88mm AA

b. Classification of Soviet Weapon Types

(1) Tanks used in the study:

- KV-1, KV-2, M-3, MK-2/3
- MK-4, T-34, T-60, T-70

(2) Artillery used in the study:

- SU-122, 122mmGun, 122mmHow
- 152mmGun, SU-152, 203mmHow

The summation of each artillery weapon type defined in Table 5-1 from the OH weapon numbers [Ref. 19:Gewpns, worksheet GEOHWPNS] do not match the number of the artillery weapon groups shown in Figure 5-5 [Ref. 19:Data98-7, worksheet CHAP5]. In this study, we used the number obtained from the summation of each artillery weapon type.

(3) APC used in the study:

- BA-64, BA-10
- Armtpt, Bren

(4) Rocket Launchers used in this study:

- BM-13

(5) Heavy antitank (ATH) weapons used in this study:

- 57mm AT, 76mmGun
- 85mm AT

The summation of each ATH weapon type defined in Table 5-1 from the OH weapon numbers [Ref. 19:Gewpns, worksheet GEOHWPNS] do not match the number of ATH weapon groups shown in Figure 5-5 [Ref. 19:Data98-7, worksheet CHAP5]. In this study, we used the number obtained from the summation of each ATH weapon type.

(6) Mortars (MTR) used in this study:

- 50mmmtr, 82mm mtr
- 120mm mtr

(7) Light antitank (ATH) weapons used in this study:

- 45mmAT
- ATR

(8) Flame/MG used in this study:

- Flame, MG
- AA MG

(9) Antiaircraft (AA) weapons used in this study:

- 25mmAA, 37mmAA
- 40mmAA, 85mmAA

The summation of each AA weapon type in Table 5-1 from the OH weapon numbers [Ref. 19:Gewpns, worksheet GEOHWPNS] do not match the number of AA weapon groups given in Figure 5-5 [Ref. 19:Data98-7, worksheet CHAP5]. In this study, we used the number obtained from the summation of each AA weapon type.

3. Unit Activities and Combat Postures

One of the interesting aspects of this battle is the engagement percentages of the forces. The Germans had a considerably larger portion of their forces in contact. This suggests that the German force may have been subjected to more fatigue than the Soviets. With an average of 97 percent of its heavy mechanized force on the front lines, the Germans, unlike the Soviets, had no reserves to use [Ref. 14:p. 9-4].

The KDB records the daily contact status and combat posture of each of the 17 German line units and each of the 48 Soviet line units. Units in contact with the enemy are further partitioned into those in active contact (fighting) and those that are in contact but not in active contact. Depending on this contact status, this study extracts three different data sets from the data on the Battle of Kursk in the KOSAVE report.

The first data set, all combat units data (ACUD), includes all combat units: contact, out of contact, active, and inactive, including HQ above division level.

The second data set, contact combat units data (CCUD), includes only combat units those are in contact with the enemy: units in contact fighting and not fighting, HQ above division level is excluded [Ref. 14:p. 5-9].

The third data set, fighting combat units data (FCUD), includes only combat units that are in contact and fighting with the enemy: HQ above division level are excluded [Ref. 14:p. 5-9].

The reason for setting these data sets is to search for the influences of the unit's contact status on the attrition rates. Since the vast majority of the Soviet units were not engaged directly with the enemy, they were not supposed to have as many casualties as the German units suffered.

The KDB also records the combat postures of units; attack, delay, hasty defense, prepared defense, and other (usually denoting reserve/transit status). Appendix E [Ref. 14] shows the daily contact and combat postures for each line unit of each force.

The majority of the German units were continually attacking from 5 July through 12 July, but almost all were in prepared defense by 17 July. The vast majority of Soviet line units were in defense postures each day except 12 July, when the Soviets counterattacked. The temporary success of the Germans in meeting the 12 July Soviet counterattack is evident in the shift of the vast majority of attacking Soviets units to hasty defense postures by 14 July [Ref. 14:p. 3-8].

The following figures show the unit status and postures for both sides. Figure 3.2 presents the percentages of line units in each force, which were in contact with the enemy. Figure 3.3 shows the percentages of line units in each force, which were attacking. In all models, the attacking side is defined according to the percentages depicted in Figure 3.3. Until day 9 (12 July), it is clear that the Germans were the attacking side. This is also supported by historical facts.

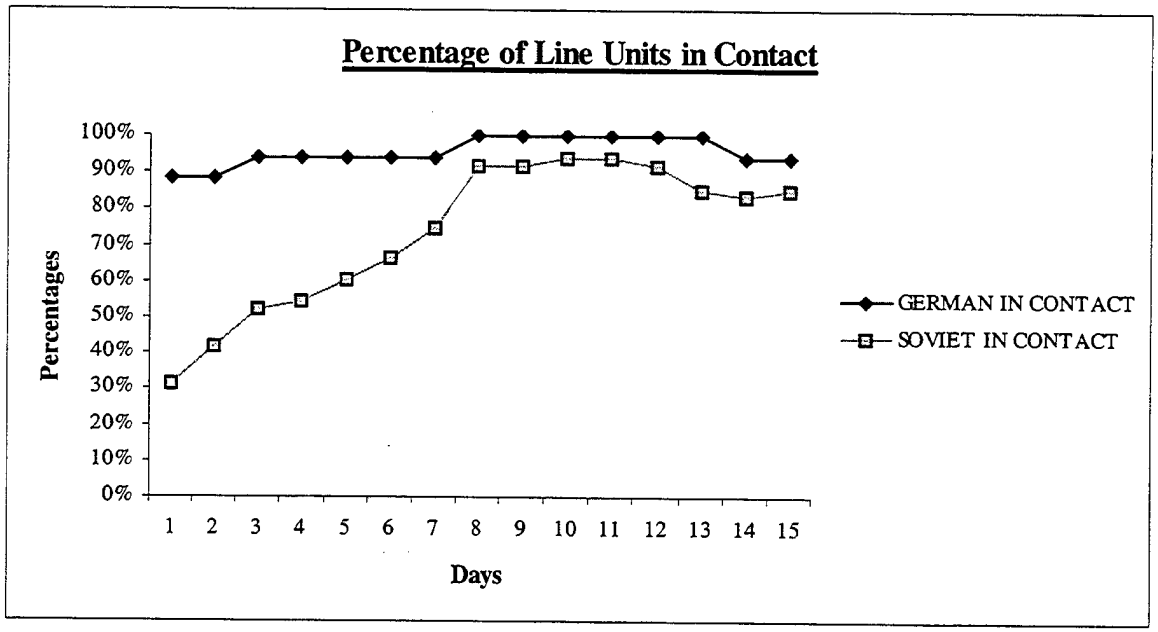

Figure 3.2. Line Units in Contact. The vast majority of German units are in contact each day while the Soviets have an increasing percentage.

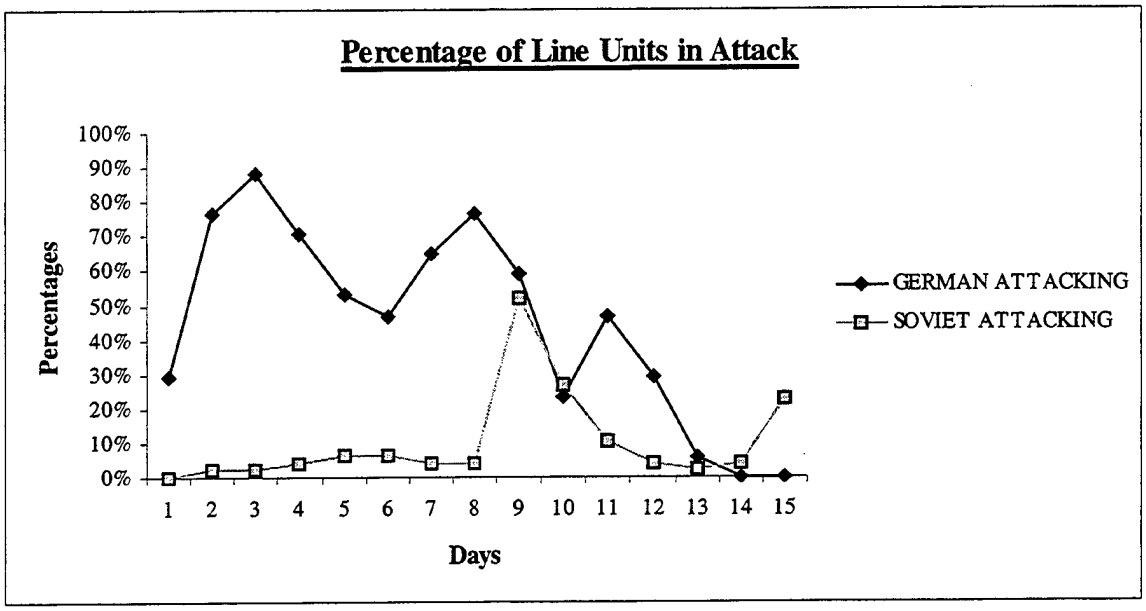

Figure 3.3. Line Units Attacking. Except for days 10, 14, and 15, the Germans have a higher attacking percentage. Although on day 9 (12 July), the Germans have a slightly higher ratio, based on historical attributes, it is assumed that the Soviets are attacking on this day.

Although on day 9 (12 July), the Germans had a slightly higher ratio of attacker units, based on the historical attributes, it is assumed that the Soviets were attacking on this day [Ref. 14:p. 3-7] [Ref. 16:p. 197] [Ref. 21:p. 225]. Although they have almost the same percentage on day 10, the Soviets are defined as the attacking side. On day 13, the Germans are defined as the attacking side, and on the last 2 days the Soviets are considered to be the attacking side.

In the attrition models in which a force ratio is used, an attacking side must be determined to calculate the attrition and the advance rates. However, in reality the attacking side may not be decided in some cases, like days 10, 13, 14. In the last days, less than 20 percent of the forces are really considered as attacking. This fact should be considered in all models throughout this study.

Figures 3.3 and 3.4 show the combat posture of each force. The vast majority of the German forces are in an attacking posture for the start of the battle. After 12 July, they are almost all in a defense posture. On most days, they do not have any forces in reserve. Unlike the Germans, most Soviet units are in a defense posture, either prepared or hasty. The highest attacking unit numbers are on 12, 13, and 18 July. On almost all these days, they have reserve forces. In all models in this study, the defender's combat posture is defined according to Figures 3.4 and 3.5. In all models, the defender's posture is decided according to the percentages shown in Figures 3.4 and 3.5.

The following three sections present the personnel and weapon OH and loss numbers of the three data sets, which were extracted from the data given in the KOSAVE study.

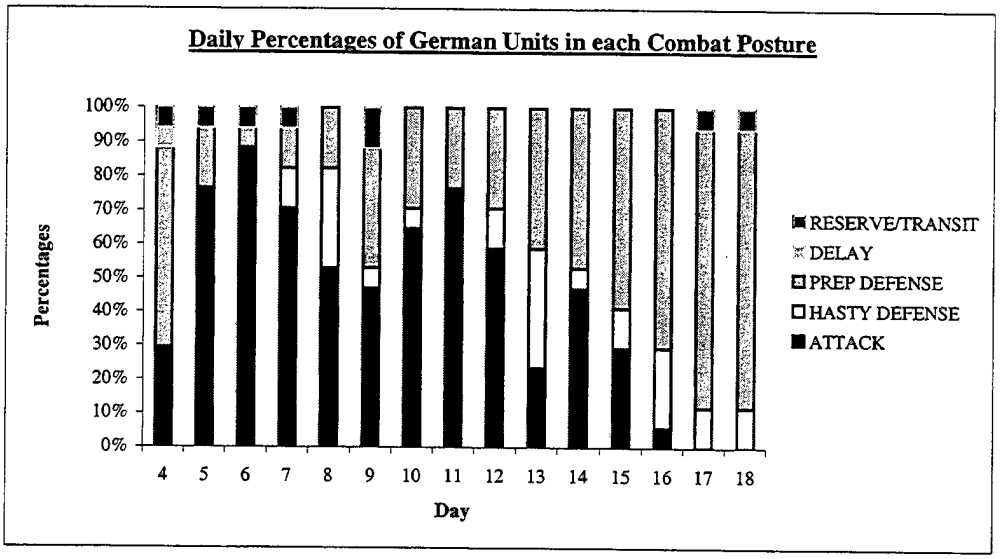

Figure 3.4. Daily Percentages of German Units in Each Combat Posture. The vast majority of units are in an attacking status.

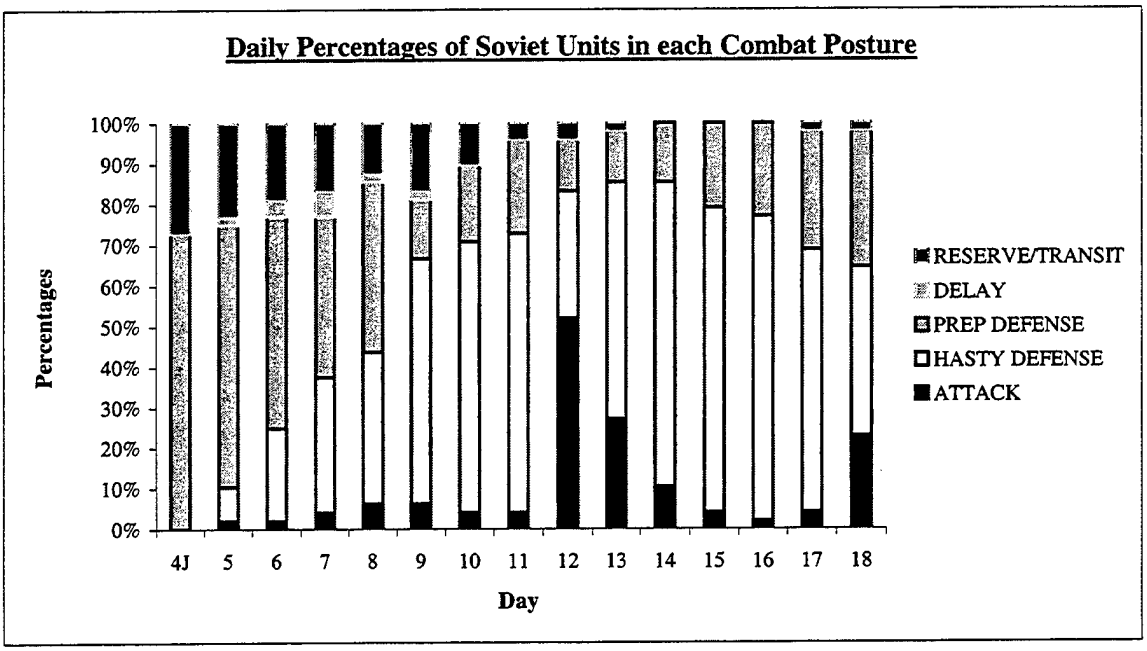

Figure 3.5. Daily Percentages of Soviet Units in Each Combat Posture. The vast majority of units are in prepared in a hasty defense posture.

4. All Combat Units Data (ACUD)

All combat units data (ACUD) includes all units; contact, out of contact, active, inactive, and including HQ above division level. The two tables below show the OH data.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	307365	1178	1277	1230	340	665	1550	494	1970	1055
2	301341	986	1254	1199	340	618	1489	488	1938	1048
3	297205	749	1249	1185	338	590	1483	483	1931	1045
4	293960	673	1237	1158	338	549	1479	481	1917	1022
5	306659	596	1296	1141	338	586	1623	525	2063	1051
6	303879	490	1296	1129	338	590	1596	518	2057	1051
7	302014	548	1281	1172	335	595	1587	514	2055	1047
8	300050	563	1286	1161	335	584	1585	513	2027	1058
9	298710	500	1268	1157	334	572	1579	506	2019	1045
10	299369	495	1262	1155	333	548	1591	499	2012	1057
11	297395	480	1258	1152	333	547	1584	487	2009	1043
12	296237	426	1258	1148	331	550	1576	481	1995	1039
13	296426	495	1255	1151	330	560	1575	470	1989	1047
14	296350	557	1258	1154	330	558	1574	471	1988	1050
15	295750	588	1250	1156	323	561	1573	465	1978	1059

Table 3.1. Daily German Onhand Personnel and Weapon Data. All Combat Units are included.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	510252	2500	718	511	319	2363	7374	17091	6612	704
2	507698	2396	705	507	315	2243	7163	16407	6408	704
3	498884	2367	676	501	312	2167	6883	15727	6119	712
4	489175	2064	661	490	308	2046	6633	15077	5787	680
5	481947	1754	648	477	309	1923	6458	14537	5523	657
6	470762	1495	640	458	308	1837	6244	14051	5274	652
7	460808	1406	629	463	307	1775	5986	13449	5027	650
8	453126	1351	628	462	304	1698	5782	12964	4850	648
9	433813	977	613	432	302	1628	5498	12245	4583	644
10	423351	978	606	424	298	1591	5356	11851	4434	644
11	415254	907	603	418	295	1587	5202	11542	4230	643
12	419374	883	601	417	295	1539	5091	11301	4134	643
13	416666	985	600	417	293	1571	4916	11093	3997	644
14	415461	978	602	417	291	1574	4933	11079	3972	645
15	413298	948	591	409	288	1542	4902	10967	3850	645

Table 3.2. Daily Soviet Onhand Personnel and Weapon Data. All Combat Units are included.

Tables 3.3 and 3.4 show daily personnel and weapon losses.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	**657**	**4**	**1**	**0**	**0**	**0**	**0**	**0**	**1**	**0**
2	6049	198	25	32	0	50	61	6	32	8
3	4157	248	5	14	2	29	6	5	7	3
4	3271	121	12	27	0	42	4	2	14	23
5	2780	108	13	17	0	12	4	5	6	14
6	2793	139	6	14	0	27	27	7	6	12
7	1879	36	18	42	3	14	9	5	2	22
8	2306	63	17	17	0	25	2	5	28	13
9	2450	98	18	11	1	28	6	8	8	12
10	1893	57	8	3	1	30	5	7	7	21
11	1985	46	7	6	0	7	7	12	3	14
12	1163	79	3	5	2	16	8	8	14	4
13	1161	23	4	1	1	5	1	11	6	4
14	786	7	2	1	0	13	1	3	1	1
15	887	6	12	5	7	9	4	8	10	9
TOTAL	**34217**	**1233**	**151**	**195**	**17**	**307**	**145**	**92**	**145**	**160**

Table 3.3. Daily German Personnel and Weapon Losses. Notice that on the first day the losses are very small. Almost all losses show a descending pattern indicating that the intensity of the battle is decreasing.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	**114**	**0**	**0**	**0**	**0**	**0**	**2**	**1**	**1**	**0**
2	8445	105	13	4	4	120	211	682	204	0
3	9325	117	30	6	3	79	286	707	293	2
4	10339	259	15	11	4	119	245	635	328	24
5	9421	315	14	13	3	113	175	501	264	24
6	11625	289	9	19	1	97	202	526	248	6
7	10724	157	13	3	1	65	267	586	251	3
8	7697	135	7	4	3	87	200	502	187	2
9	**19363**	**414**	**16**	**30**	**2**	**84**	**289**	**723**	**290**	**3**
10	10470	117	10	8	4	45	162	387	153	0
11	8674	118	5	8	5	46	155	334	185	1
12	4038	96	5	1	0	62	123	250	99	0
13	2905	27	3	0	2	14	182	239	146	0
14	1182	42	0	2	2	1	12	34	29	0
15	3230	85	4	8	4	30	64	163	120	0
TOTAL	**117552**	**2276**	**144**	**117**	**38**	**962**	**2575**	**6270**	**2798**	**65**

Table 3.4. Daily Soviet Personnel and Weapon Losses. Notice that on the first day losses are very small, like the German losses. Almost all losses show a descending pattern except for the ninth day of the battle. This day is the bloodiest tank battle in history.

5. Contact Combat Units Data (CCUD)

CCUD includes only combat units in contact with the enemy: units in contact fighting and not fighting, HQ above division level is excluded [Ref. 14:p. 5-9].

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	265823	942	1136	1207	310	630	1372	471	1842	905
2	262055	965	1123	1182	310	583	1329	465	1811	898
3	276383	731	1194	1168	320	588	1479	482	1929	997
4	273660	652	1182	1141	338	547	1475	480	1915	974
5	275511	564	1212	1124	338	557	1471	475	1909	1030
6	287391	389	1205	962	320	545	1506	485	1853	984
7	248538	525	1170	1155	335	566	1435	464	1901	984
8	279722	563	1244	1144	281	582	1581	512	2025	1007
9	279046	483	1227	1140	334	570	1575	505	2017	994
10	279697	495	1233	1138	333	546	1587	498	2010	964
11	276604	474	1229	1135	333	541	1576	486	2007	950
12	291571	418	1229	1131	331	543	1568	480	1993	946
13	289582	480	1212	1134	330	553	1555	469	1987	954
14	237336	441	1126	957	315	501	1486	420	1790	812
15	235653	472	1152	959	305	484	1485	414	1780	788

Table 3.5. Daily German CCUD Onhand Personnel and Weapon Data.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	138378	129	184	11	96	614	2494	5393	1798	9
2	181474	396	211	61	142	726	3000	6228	2315	47
3	221666	1006	209	235	150	845	3230	6563	2556	84
4	238993	980	228	234	125	851	3165	6645	2479	106
5	256687	742	221	227	148	966	3507	7198	2580	113
6	284050	830	239	261	148	997	3660	7589	2736	160
7	297105	869	269	312	126	1066	4013	8214	3077	150
8	358172	1158	331	420	146	1257	4942	9911	4048	191
9	344513	832	339	353	122	1216	4939	9848	3998	168
10	339299	875	342	414	164	1206	4829	9613	3923	184
11	330225	784	340	403	161	1164	4673	9335	3721	174
12	302666	715	337	352	139	1127	4454	8915	3573	149
13	272394	573	330	291	109	1041	4140	8375	3352	127
14	263878	569	313	291	59	1020	4073	8093	3259	127
15	282532	624	318	333	65	1022	4102	8050	3184	152

Table 3.6. Daily Soviet CCUD Onhand Personnel and Weapon Data.

Tables 3.7 and 3.8 show personnel and weapon losses.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	651	0	1	0	0	0	0	0	1	0
2	5823	180	19	32	0	50	61	6	31	8
3	4133	240	3	14	2	29	6	5	7	3
4	3249	113	12	27	0	42	4	2	14	23
5	2746	108	13	17	0	12	4	5	6	14
6	2677	102	5	14	0	26	27	7	6	11
7	1851	36	18	42	3	14	9	5	2	22
8	2294	63	17	17	0	25	2	5	28	13
9	2435	92	17	11	1	28	6	8	8	12
10	1880	57	8	3	1	30	5	7	7	21
11	1966	41	9	6	0	6	7	12	3	14
12	1151	79	3	5	2	16	8	8	14	4
13	1150	22	4	1	1	5	1	11	6	4
14	734	4	2	1	0	9	1	3	1	1
15	866	5	11	5	7	8	4	8	10	9
TOTAL	**110016**	**2053**	**110**	**96**	**21**	**699**	**2512**	**5807**	**2691**	**16**

Table 3.7. Daily German CCUD Personnel and Weapon Losses.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	104	0	0	0	0	0	2	1	1	0
2	8227	73	8	4	2	108	209	644	204	0
3	8880	101	25	6	1	64	277	660	288	2
4	9001	255	9	11	1	90	230	562	309	6
5	7907	300	9	11	0	77	156	411	223	3
6	10678	228	9	7	0	70	202	496	239	1
7	10195	116	10	3	0	52	266	566	247	1
8	7444	125	5	3	2	56	200	473	184	0
9	**18884**	**392**	**16**	**25**	**2**	**63**	**283**	**671**	**290**	**2**
10	10169	110	6	7	4	31	162	363	151	0
11	8390	114	4	8	5	33	152	310	180	1
12	3841	93	2	1	0	16	115	230	99	0
13	2781	27	3	0	2	14	182	239	146	0
14	1120	34	0	2	2	1	12	29	19	0
15	3170	85	4	8	0	24	64	152	111	0
TOTAL	**110791**	**2053**	**110**	**96**	**21**	**699**	**2512**	**5807**	**2691**	**16**

Table 3.8. Daily Soviet CCUD Personnel and Weapon Losses.

6. Fighting Combat Units Data (FCUD)

FCUD includes only combat units in contact and fighting with the enemy: HQ above division level are excluded [Ref. 14:p. 5-9].

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	97740	290	437	318	61	241	604	150	671	241
2	247866	965	1064	1182	310	557	1224	434	1727	898
3	261368	731	1131	1168	308	555	1326	461	1804	985
4	211212	652	921	1141	272	423	926	400	1476	826
5	227314	564	1014	1124	284	468	1075	417	1598	864
6	224664	389	947	962	284	434	1023	369	1439	816
7	200686	525	985	1155	299	490	1104	398	1639	828
8	232938	563	1060	1144	281	506	1250	444	1763	851
9	262920	483	1176	1140	334	542	1439	491	1936	994
10	279697	495	1233	1138	333	546	1587	498	2010	964
11	208498	415	955	970	232	392	1141	392	1539	627
12	226075	356	1027	966	250	437	1300	399	1630	755
13	131800	193	540	514	140	214	664	214	788	416
14	149538	363	640	792	204	249	640	250	982	599
15	188079	352	891	744	228	360	1094	323	1333	622

Table 3.9. Daily German FCUD Onhand Personnel and Weapon Data.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
2	84783	83	126	10	118	334	1309	3093	1119	9
3	141589	605	147	175	150	554	1813	4040	1635	70
4	163378	980	157	232	77	541	1972	4369	1741	106
5	145875	646	112	221	78	489	1729	3720	1473	104
6	179607	352	162	162	103	674	2421	5061	1796	71
7	166526	483	139	163	126	586	1971	4107	1462	111
8	219343	480	202	201	76	761	3087	5999	2369	125
9	252844	525	262	231	115	914	3698	7275	3012	128
10	175121	349	213	114	44	688	2970	5768	2352	63
11	206465	513	204	293	117	720	2947	6006	2415	123
12	89898	68	113	16	48	317	1507	2806	1193	13
13	87769	76	124	16	24	336	1380	2887	1178	14
14	37981	108	36	16	0	136	631	1174	475	29
15	119346	408	127	176	65	438	1725	3269	1350	114

Table 3.10. Daily Soviet FCUD Onhand Personnel and Weapon Data.

Tables 3.11 and 3.12 show personnel and weapon losses.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	607	0	1	0	0	0	0	0	1	0
2	5736	180	19	32	0	50	61	6	31	8
3	3471	240	3	14	2	29	3	4	7	3
4	2933	113	12	27	0	42	4	2	11	23
5	2622	108	13	17	0	10	4	4	3	14
6	2516	102	5	14	0	25	12	5	5	11
7	1723	36	17	42	3	14	5	4	2	22
8	2217	63	11	17	0	24	2	5	28	13
9	2426	92	17	11	1	28	6	8	8	12
10	1880	57	8	3	1	30	5	7	7	21
11	1553	41	3	6	0	6	3	10	2	14
12	934	75	2	5	0	10	8	6	14	4
13	387	13	4	0	0	3	1	4	6	1
14	418	4	1	1	0	4	0	2	1	0
15	699	5	8	4	6	5	3	3	10	9
TOTAL	**30122**	**1129**	**124**	**193**	**13**	**280**	**117**	**70**	**136**	**155**

Table 3.11. Daily German FCUD Personnel and Weapon Losses.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/MG	AA
1	0	0	0	0	0	0	0	0	0	0
2	8194	68	8	4	2	105	209	644	204	0
3	8797	73	25	6	1	61	277	653	288	0
4	8840	255	7	11	1	85	230	547	308	6
5	7493	287	9	10	0	69	150	399	193	3
6	8549	145	9	7	0	60	192	415	203	1
7	8094	108	6	3	0	41	227	496	216	1
8	6593	115	4	2	2	50	175	413	166	0
9	18042	375	15	24	2	58	261	611	255	1
10	8661	36	5	4	4	25	135	286	104	0
11	6124	99	4	6	5	24	115	209	135	1
12	2466	6	1	0	0	11	71	125	60	0
13	2105	0	3	0	0	14	173	180	115	0
14	448	6	0	0	0	1	4	14	8	0
15	2395	84	0	8	0	14	48	137	98	0
TOTAL	**96801**	**1657**	**96**	**85**	**17**	**618**	**2267**	**5129**	**2353**	**13**

Table 3.12. Daily Soviet FCUD Personnel and Weapon Losses.

D. STATISTICAL COMPARISONS OF PERSONNEL AND WEAPONS

1. Personnel Statistics

This section presents statistical comparisons of German and Soviet personnel strength and casualties in the Battle of Kursk.

a. Onhand Personnel

Figures 3.6 and 3.7 show the percentages of contact and not in contact OH personnel strength for the Germans and Soviets respectively. Almost all of the available German personnel strength was engaged after the first day (4 July). The Germans have a greater in contact ratio than the Soviets each day. The Soviet engagement fraction peaked during days 8-11 (11-14 July) at 80-84 percent and dropped to 70 percent by the last day (18 July).

From 5 through 18 July (days 2-15), percentage of German engagement never dropped below 86 percent. Since rest and replenishment were minimal in such a heavily committed force, the effectiveness and efficiency of German combat elements probably deteriorated over time relative to that of the Soviet force [Ref. 14:p. 4-2].

Figures 3.8 and 3.9 show the percentage of fighting and not fighting personnel strength for the units in contact of the Germans and Soviets respectively. The vast majority of German personnel strength is in a fighting status except for the first and thirteenth days. Unlike the Germans, the Soviets have less personnel strength in fighting status, especially on the first day.

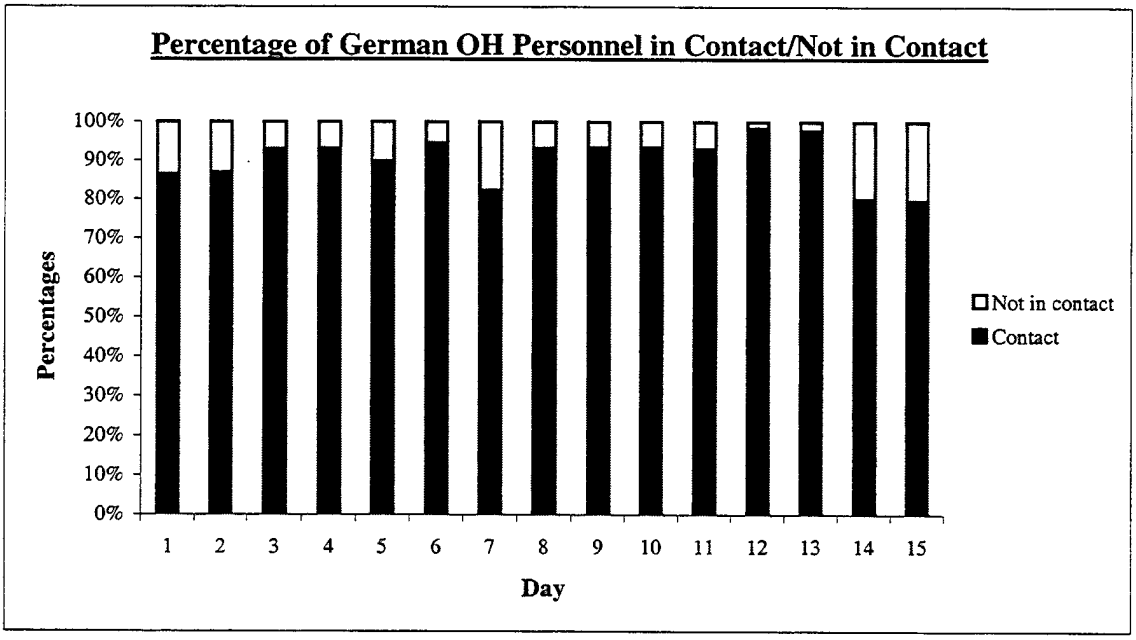

Figure 3.6. German Onhand Personnel in Contact/Not in Contact. Almost all German personnel are in contact with an average of 93 percentages.

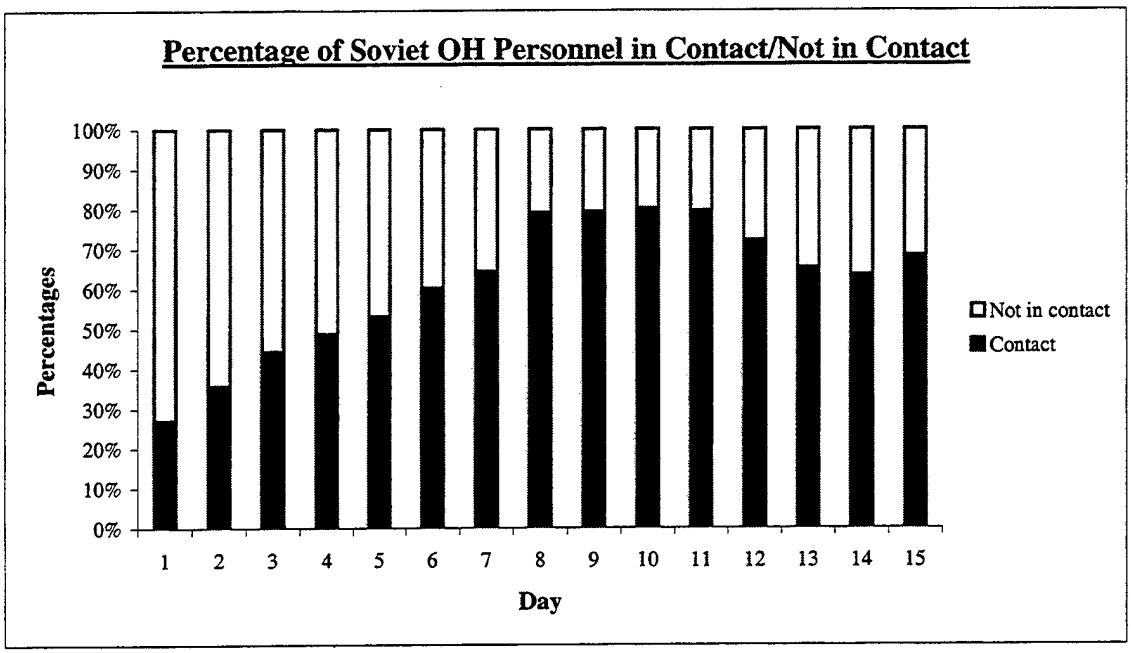

Figure 3.7 Soviet Onhand Personnel in Contact/Not in Contact. The ratio of contact personnel strength increases until the eight day.

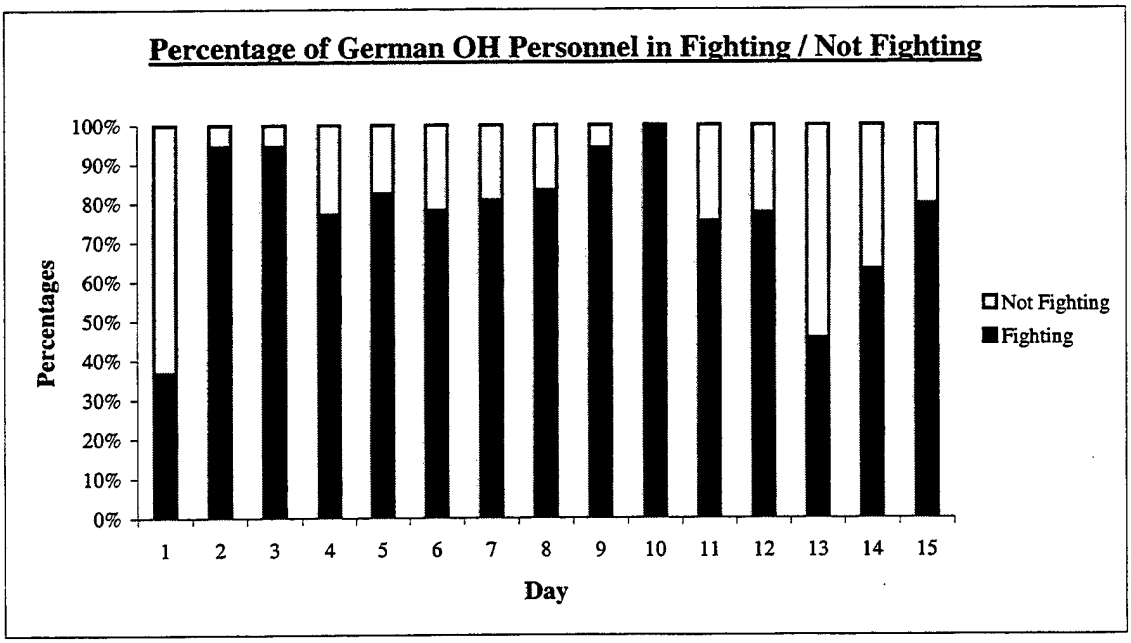

Figure 3.8. German Onhand Personnel in Fighting/Not Fighting. On the first day, the fighting personnel ratio is less than 40 percent.

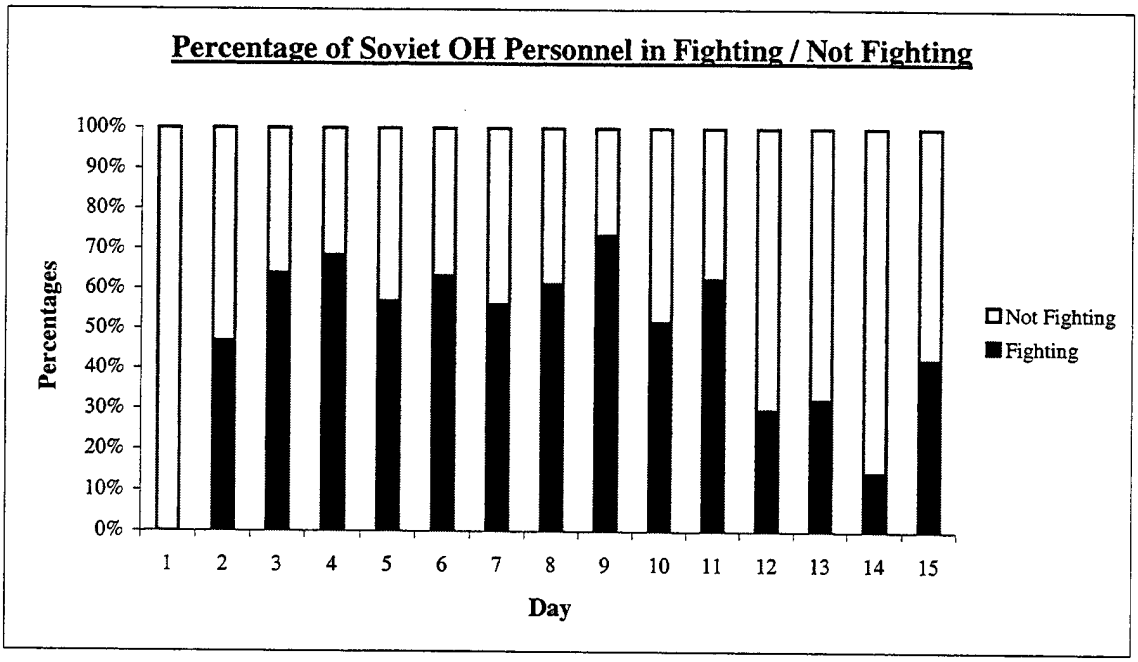

Figure 3.9 Soviet Onhand Personnel in Fighting/Not Fighting. On the first day, the Soviets were not engaged with the enemy.

b. Personnel Casualties

Figure 3.10 presents the ratio of German personnel losses in contact and not in contact combat units. All German personnel losses come from the contact combat units. Figure 3.11 shows the ratio of Soviet personnel losses in contact and not in contact combat units. Almost all Soviet personnel losses come from the contact combat units.

Figure 3.12 presents the German personnel loss ratios for the Fighting/Not Fighting combat units out of contact units. Nearly all of the German personnel losses come from the fighting units, although for days 13 and 14, the ratio is slightly lower than the other days. However, this not an unexpected result, because still the rest of the losses come from the contact combat units. Like Figure 3.12, Figure 3.13 presents the Soviet personnel loss ratios for the Fighting/Not Fighting combat units out of contact units.

Since the Soviets do not have any units in fighting position, they did not suffer any casualties on the very first day. Historical accounts support the fact that the Kursk offensive did not effectively begin until July 5, 1943.

Figures 3.14 and 3.15 show the comparison of the German and the Soviet personnel losses for ACUD and FCUD respectively. Both figures show that the Soviets suffered more casualties than the Germans each day. The Germans suffered many more personnel losses at the beginning of the battle when they attacked the heavily mined and fortified Soviet defense. The peak Soviet personnel losses occur on the ninth day (12 July) of the battle.

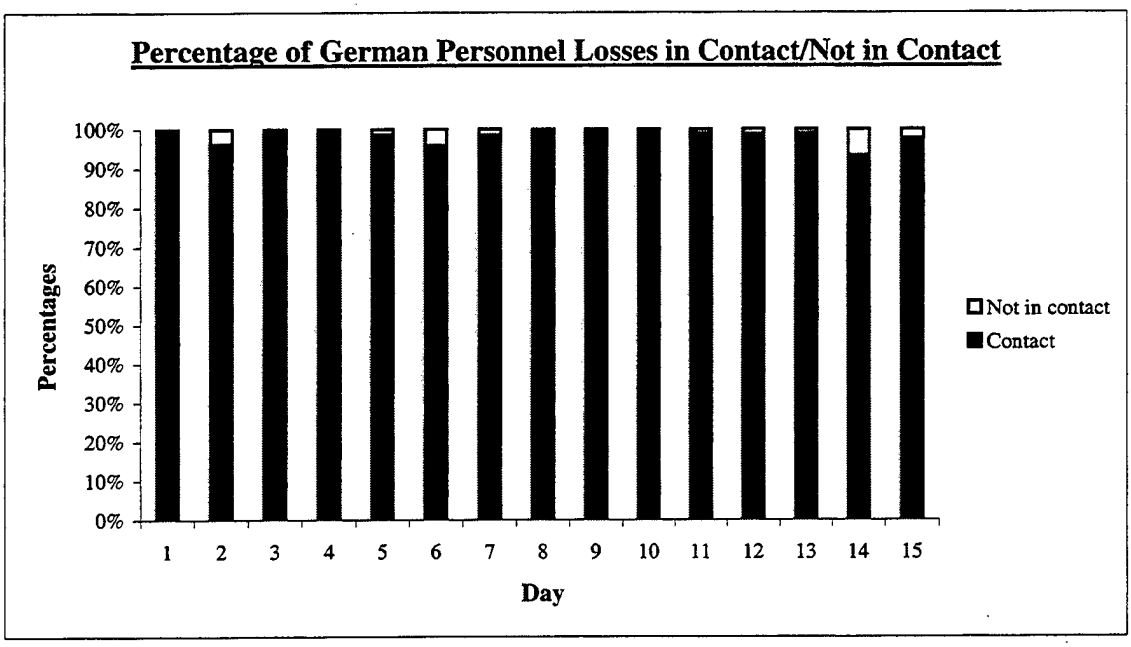

Figure 3.10. German Personnel Losses in Contact/Not in Contact. Almost all German personnel losses come from the contact units.

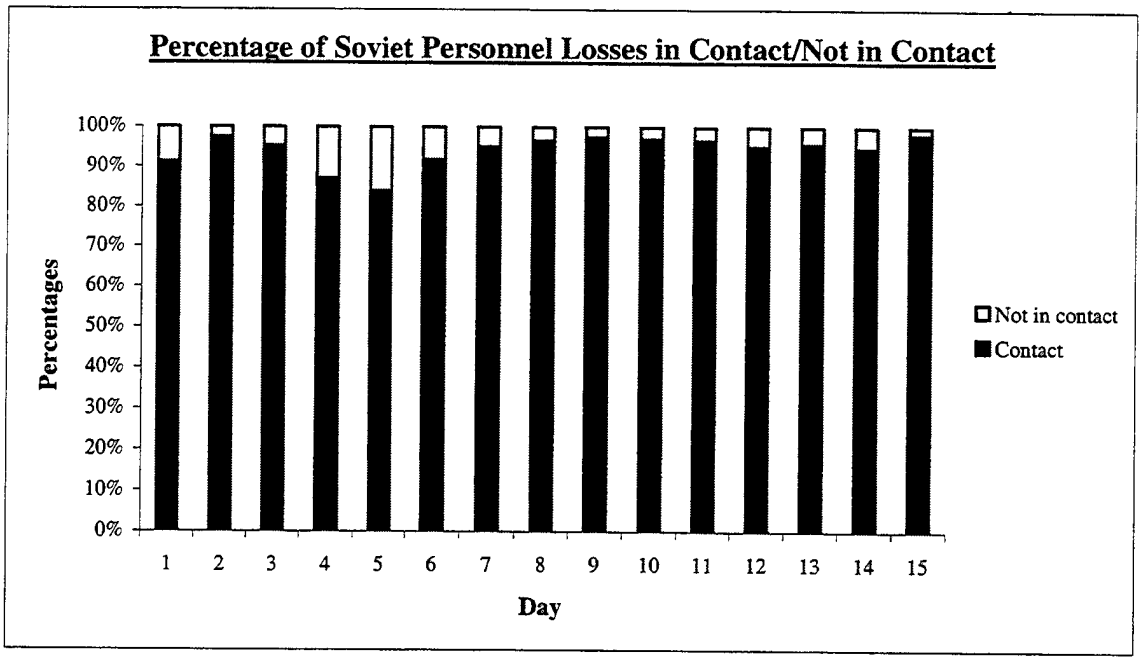

Figure 3.11 Soviet Personnel Losses in Contact/Not in Contact. The majority of the Soviet personnel losses come from the contact units.

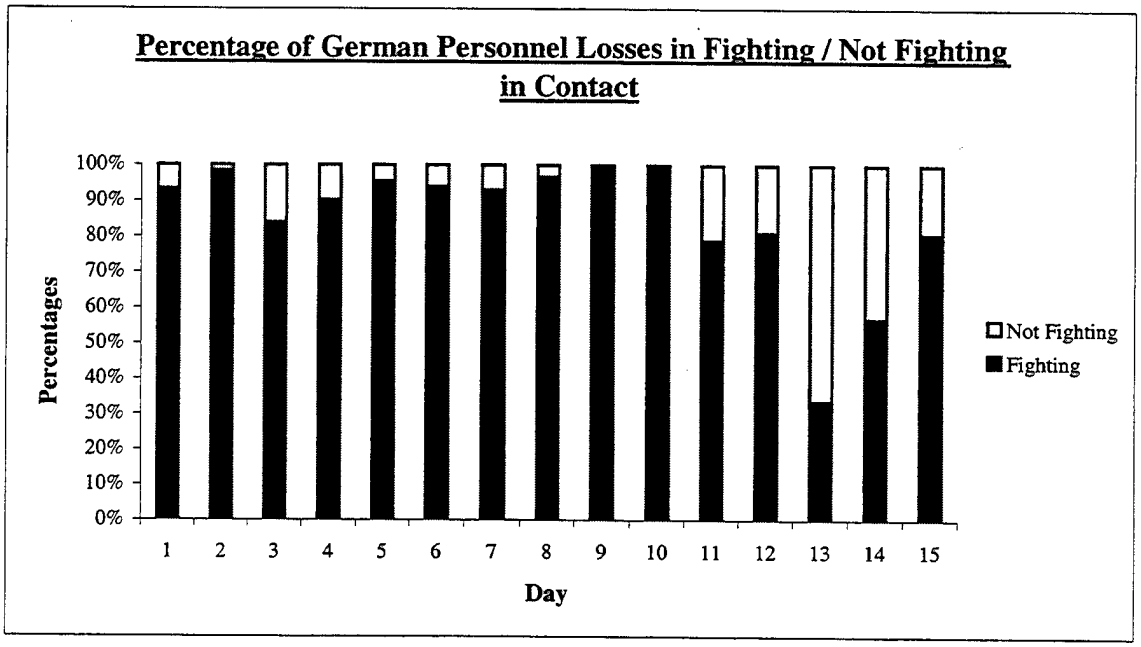

Figure 3.12. German Personnel Loss Ratios for Fighting/Not Fighting Combat Units in Contact. Except for the 13th and 14th days, almost all the German losses come from the fighting units.

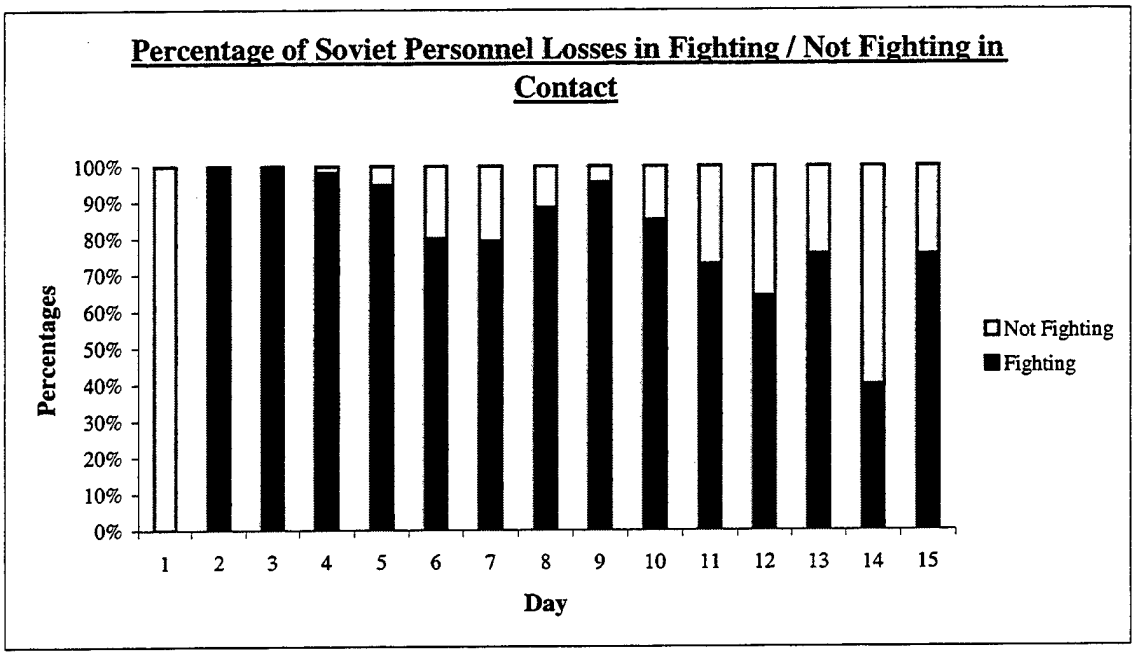

Figure 3.13. Soviet Personnel Loss Ratios for Fighting/Not Fighting Combat Units in Contact. The Soviets do not have any losses in the first day for the Fighting units.

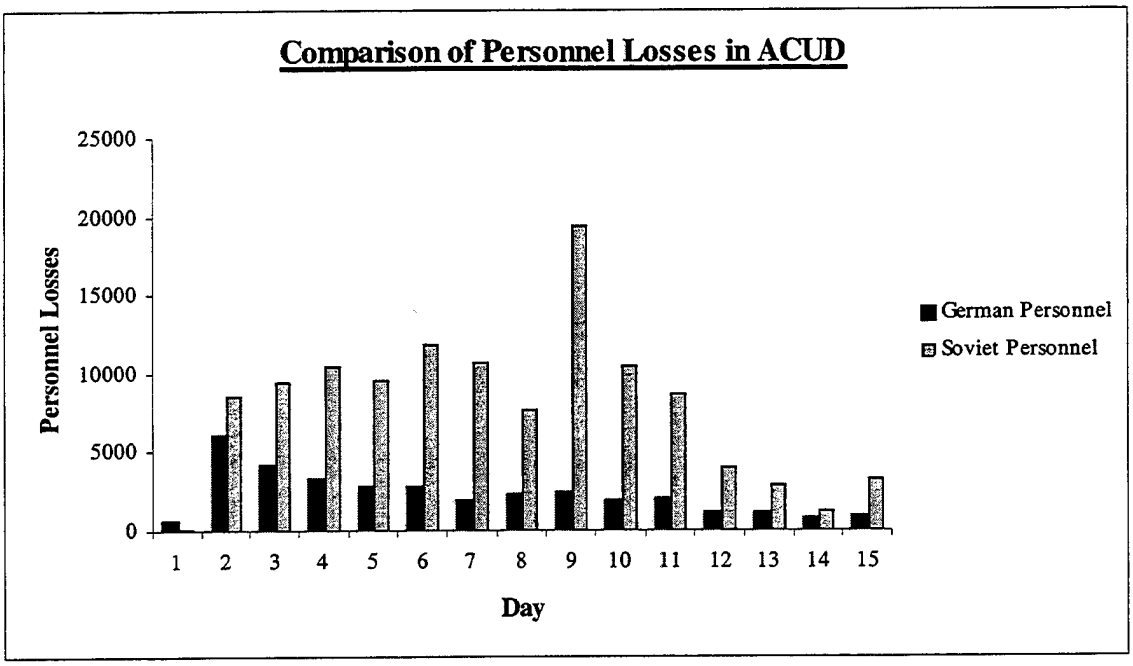

Figure 3.14. Comparison of Daily Personnel Losses in ACUD for Both Forces.

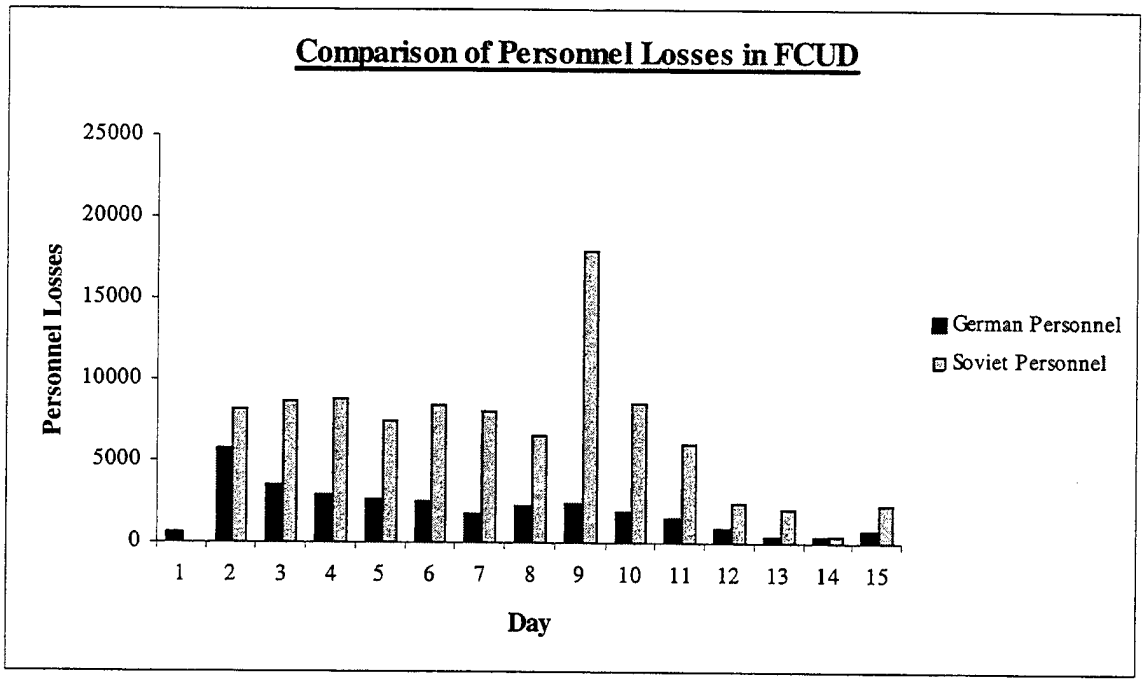

Figure 3.15. Comparison of Daily Personnel Losses in FCUD for Both Forces.

2. Tank Statistics

This section presents statistical comparisons of German and Soviet tank strength and casualties during the campaign for the purpose of providing insight into the Battle of Kursk regarded as, perhaps, the greatest armor battle in history.

a. Onhand Tanks

Figures 3.16 and 3.17 show the percentages of contact and not in contact OH tank numbers for the Germans and the Soviets respectively. Almost all of the German tanks were engaged during the campaign. The Germans have a greater in contact ratio than the Soviets in all days. The Soviet engagement fraction peaked during days 8-12 (11-15 July) with an average of 90 percent.

Figures 3.18 and 3.19 show the percentage of fighting and not fighting OH tank numbers in contact for the Germans and the Soviets respectively. The vast majority of German personnel strength is in a fighting status except for the first and thirteenth days as in the personnel strength explained in Section 1.

From Figure 3.19, it is clear that the Soviets have very small percentages of their tanks in contact with the enemy except on the fourth day. The peak day for the Soviets is 7 July (day 4) in which all Soviets tanks were engaged with the enemy. They do not have any tanks engaged on the first day and a very small percentage on days 12, 13, and 14. Remember that both fighting and not fighting units are in contact status.

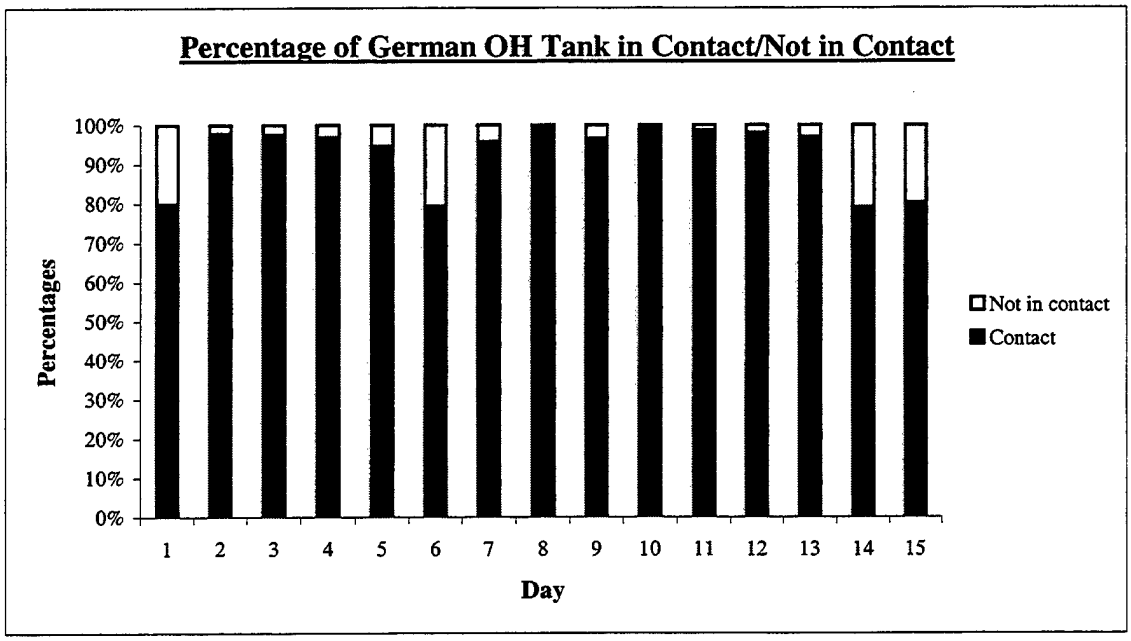

Figure 3.16. German Onhand Tanks in Contact/Not in Contact. Almost all German personnel are in contact with an average of 90 percent.

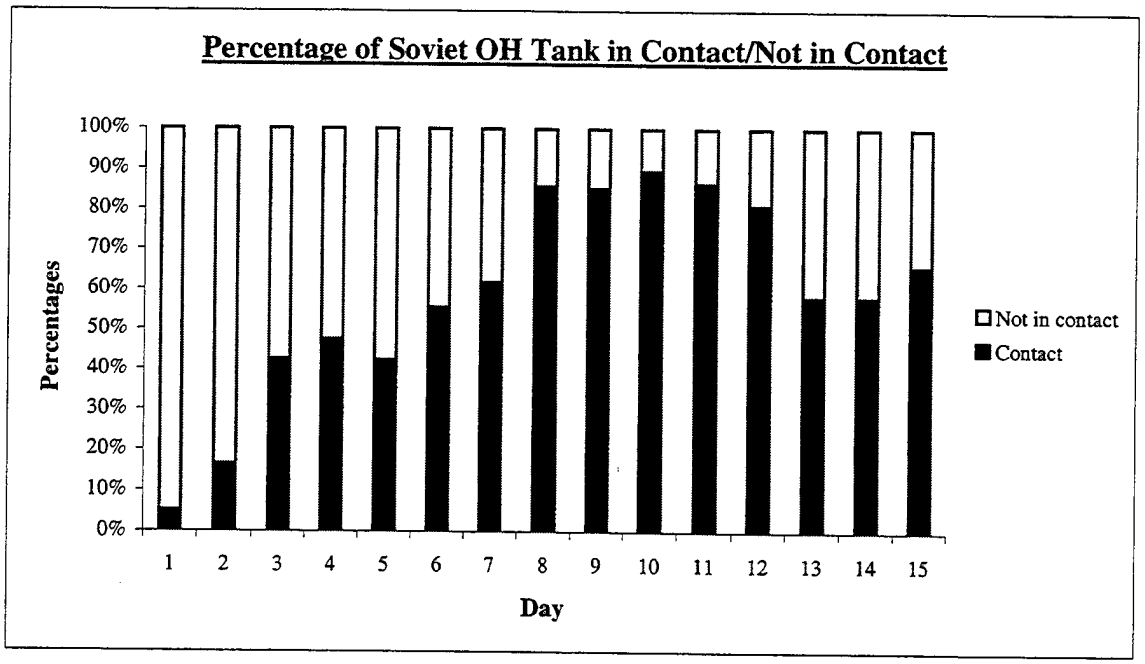

Figure 3.17. Soviets Onhand Tanks in Contact/Not in Contact. The ratio of contact tank numbers increase until the eight day.

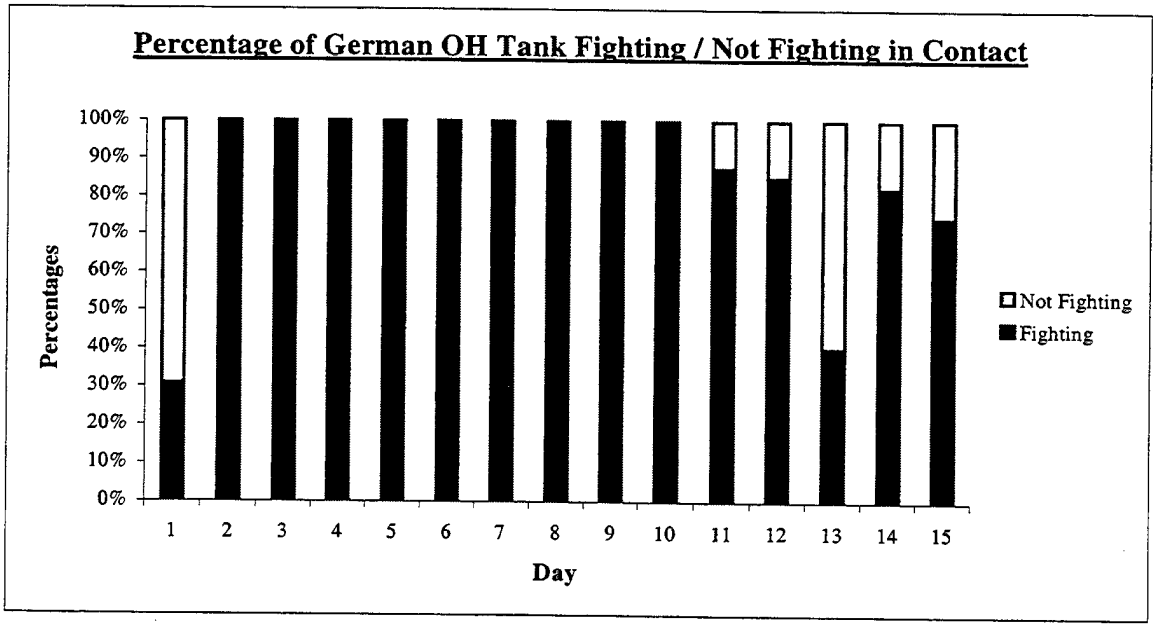

Figure 3.18. German Tank Strength Ratios for Fighting/Not Fighting Combat Units in Contact. On days 1 and 13, the fighting tank strength ratio is less than 40 percent.

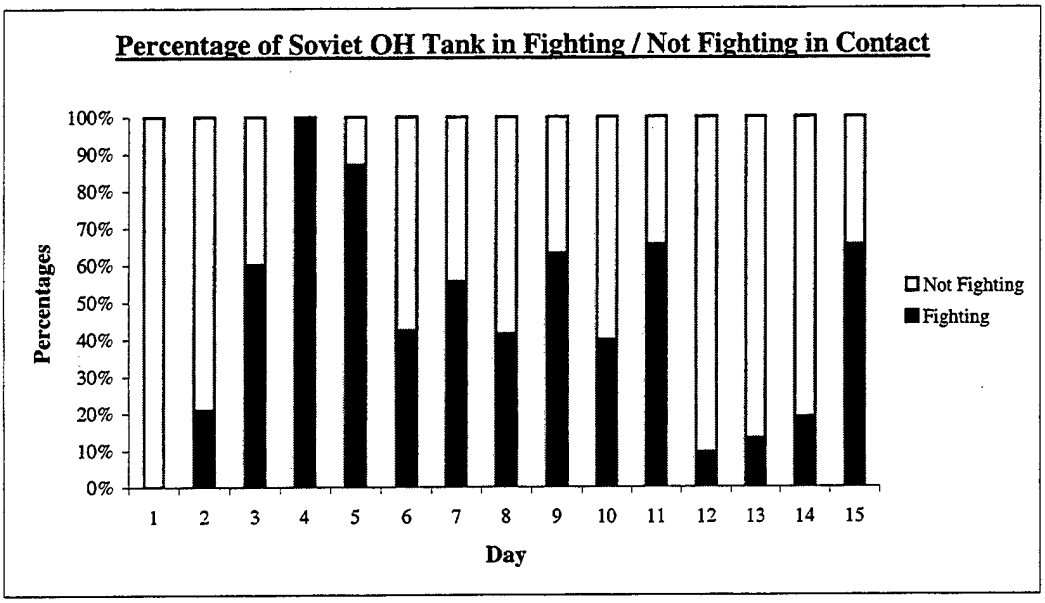

Figure 3.19. Soviet Tank Strength Ratios for Fighting/Not Fighting Combat Units in Contact. On the first day, all Soviet tank strength is in not fighting status.

Figure 3.20 presents the comparison of the number of tanks in ACUD for the Soviets and Germans. The Soviets have tank superiority over the Germans during the battle. Figure 3.21 shows that when the fighting units are considered, the Germans almost have the same number of tanks, even though on some days they have more tanks than the Soviets.

b. Tank Losses

Kursk has been cited as the largest tank battle in history. This is especially true for the battle at Prokhorovka (on July 12), which was the turning point in the Kursk campaign, in which an unprecedented number of German and Soviet tanks engaged in direct combat [Ref. 10].

Figure 3.22 shows the comparison of daily tank losses for both forces in ACUD. In the first three days, the Germans have more tank losses than the Soviets. The peak losses of the Germans happened on the second and third day (5 and 6 July). In the rest of the campaign, the Soviets suffer much more tank casualties than the Germans. On the ninth day (12 July), the Soviets lost 414 tanks. This is the heaviest loss of tanks in a battle in history.

Figure 3.23 presents the comparison of tank losses in FCUD. On the first day, both forces did not lose any tanks in active contact, which also supports the fact that the actual battle began on the second day (5 July). After the 9th day (12 July), the tank losses for both sides decreased, supporting the fact that the battle lost its intensity after July 12, 1943. The lowest number of tank losses for both forces are on the 13rd and 14th days. The trend of tank losses of the forces supported by historical accounts should be considered in the models throughout this study.

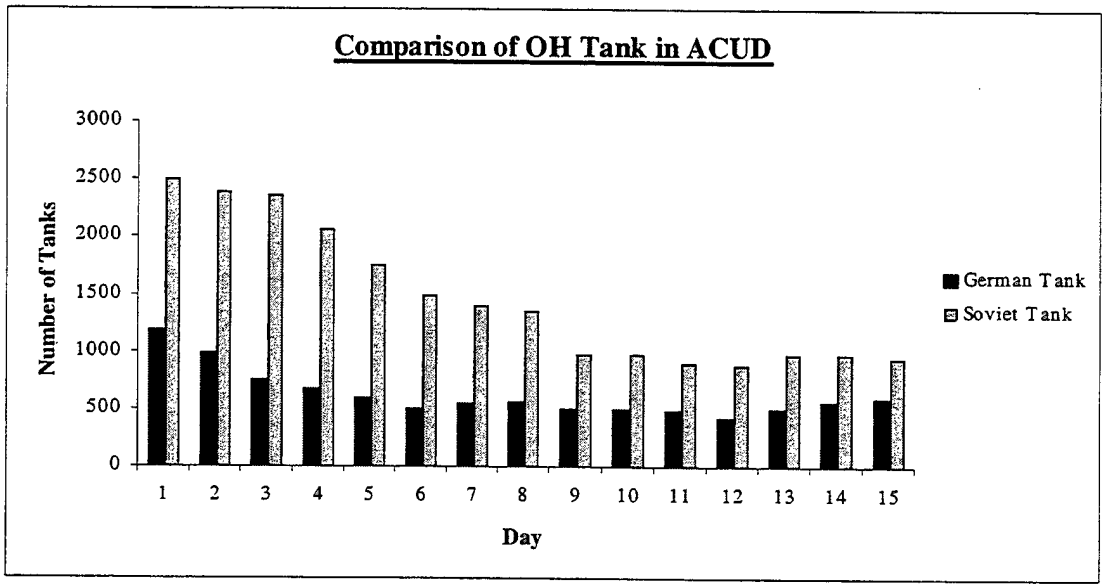

Figure 3.20. Comparison of Daily Tank Strength for the Germans and the Soviets.

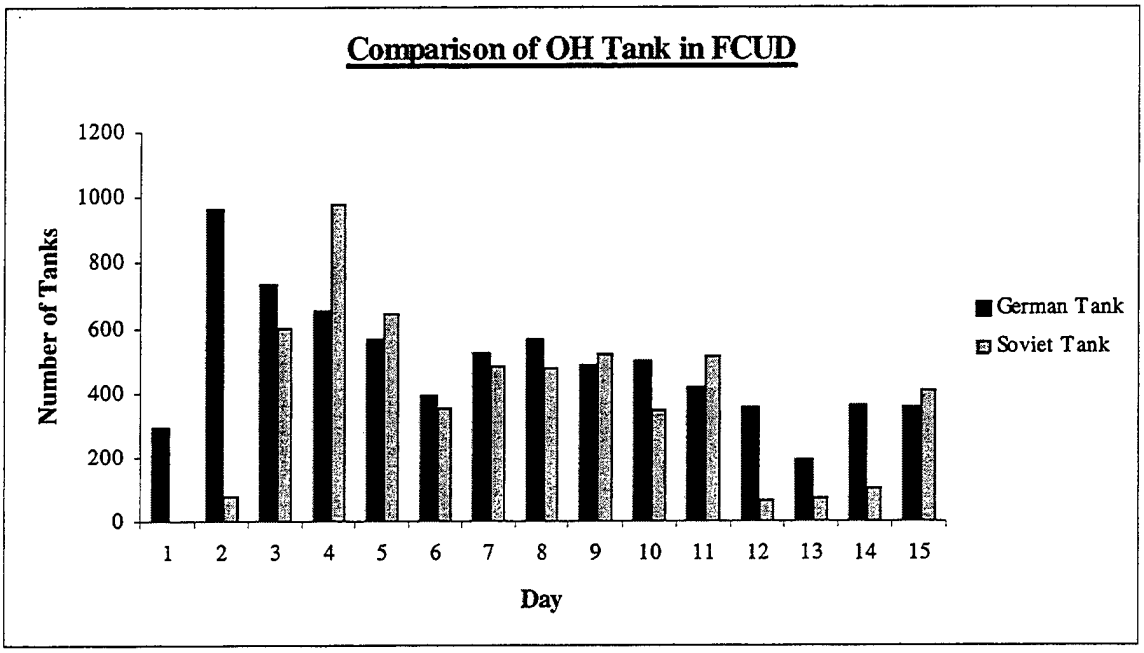

Figure 3.21. Comparison of Daily Tank Strength in FCUD for Both Forces.

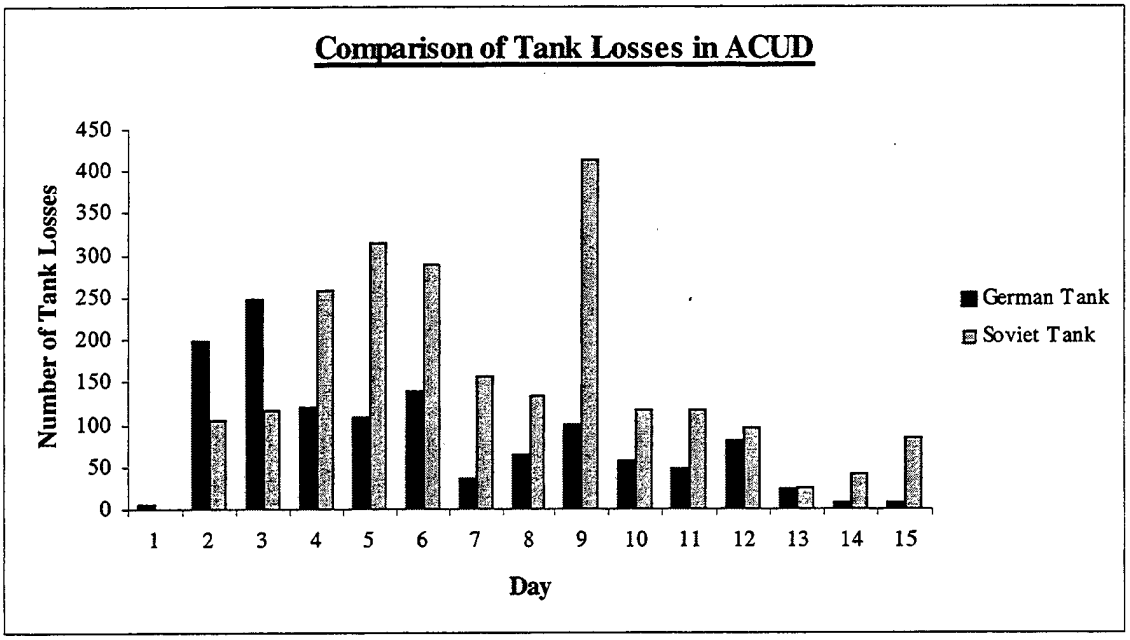

Figure 3.22. Comparison of Tank Losses of Both Forces in ACUD.

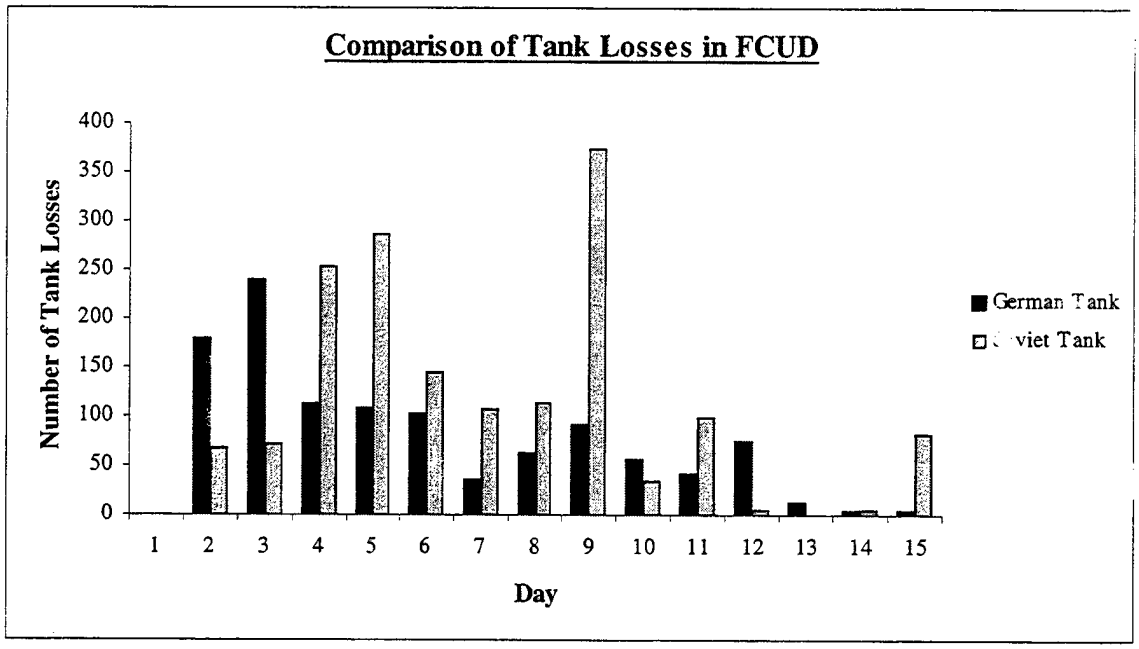

Figure 3.23. Comparison of Tank Losses of Both Forces in FCUD.

IV. APPLICATION OF DIFFERENT METHODOLOGIES TO THE DATA ON THE BATTLE OF KURSK

This chapter applies three different Firepower score approaches to the three different data sets that were extracted from the data on the Battle of Kursk. The three Firepower score algorithms are the ATLAS ground attrition, RAND's Situational Force Scoring, and Dupuy's QJM methods. For all three cases a comparison is made between the estimated losses and real losses.

A. APPLICATION OF ATLAS GROUND ATTRITION MODEL

In this section, the ATLAS ground attrition algorithm is applied to the data on the Battle of Kursk. This method is applied to three different data sets. These data sets were extracted from the data on the Battle of Kursk [Ref. 14] according to the combat engagement status, which were presented in detail in Chapter III.C.

The ATLAS ground attrition algorithm works as follows. First, all weapon types are aggregated into a single scalar measure of the combat power for each side. Then, the ratio of attacker combat power to defender combat power is computed. The ATLAS ground attrition algorithm uses the ratio of attacker combat power to defender combat power to determine the amount of combat power destroyed by the enemy. Once the combat power losses are calculated, the losses are allocated to the different weapon groups. A comparison is made between the estimated and real losses.

Section 1 explains how the firepower score values are determined. The allocation of combat power losses to different weapon categories is presented in Section 2.e. In Sections 2, 3, and 4, the ATLAS attrition algorithm is applied to each of the all combat units (ACUD), contact combat units (CCUD) and fighting combat units (FCUD) data sets. Section 5 shows how including combat power of the air sorties affect the quality of the fits. The air sortie data contains bombing and ground attack air sorties that should be directly affecting the ground attrition. In Section 6, the firepower score values used in Bracken's [Ref. 8] and Fricker's [Ref. 9] analyses of the Ardennes campaign and Clemens's [Ref. 10] and Turkes's [Ref. 11] analyses of the Battle of Kursk. Note: all these authors used the same weights.

1. Determining Firepower Score Values

Several approaches to determining firepower score values have been used over the years [Ref. 6]. None of the methods are completely satisfactory because the simple addition of scalar scores cannot capture the variety of characteristics and interactions in a complex combat environment [Ref. 6]. RAND proposed a new ground force scoring system to replace the WEI/WUV (Weapon Effectiveness Index, Weighted Unit Value system) [Ref. 7]. They categorize weapons into 13 groups, such as tanks, APCs, and mortars. These weapon groups consist of different weapon categories. As an example, the tank weapon group includes the M1-A1, M60 and M1 tank categories. Each category includes weapons that are qualitatively about 30 percent or so different from weapons in other categories within the same group. This provides sufficient homogeneity so that each weapon within a category can use the same score [Ref. 7].

Since killer-victim (KV) scoreboards are not available, there was not an objective method to use. RAND's force scoring system was the most detailed and well-documented system that we have. Thus, we use these firepower score values as a baseline to aggregate the weapon groups presented in the KDB. Since the weapons in the data on the Battle of Kursk are not the same as RAND's weapon categories (based on 1991 systems) [Ref. 7], firepower score values relative to RAND's firepower score values are determined. Table 4.1 shows RAND's weapon groups, categories and minimum and maximum scores for each weapon group. For more detailed information about the weapon categories, see [Ref. 7: p. 88]. According to the average scores, a tank is about five times more valuable than a mortar, an artillery piece is roughly four times more valuable than a short range anti-tank weapon and so on.

The KDB does not include killer-victim data, which would allow for the use of an objective Firepower score approach, such as an Anti-Potential Potential method [Ref. 6].

As explained in detail in Chapter III, the KOSAVE II [Ref. 14] report uses nine weapon groups. The firepower score values assigned to these weapon categories are presented in Table 4.2. The tanks are chosen as the base weapon category with a firepower score value of 100. Firepower score values are not assigned to each specific weapon type, rather it is assumed that all weapon types have the same score value in their weapon group. Therefore, all tank types have the same firepower score value of 100. Other weapon groups are assigned to their firepower score values relative to RAND's score proportions. For instance, mortars are five times less valuable than the tanks, so they have a firepower score value of 20.

Tanks have the highest score. In RAND's scores, rocket launcher systems are in the artillery group. In order to be consistent with the nine weapon groups defined in the KOSAVE II [Ref. 14] report, rocket launchers are considered separately. As they are more valuable than the field artillery, they have slightly higher firepower score value.

Group	Category	Minimum Score	Maximum Score	Average Score
Tanks	M1-A1, M1, M60	1.0	7.5	5
IFV/AA	M-2, BMP-1	2.5	3.5	3
ARV/AA	ITV, ATGM	1.5	2.5	2
LARV	Lgt. Veh.	0.8	0.8	0.8
APC	IFV/No AT, APC	0.8	1.3	1
LRAArm	Imp TOW/Veh	0.8	1.5	1.2
SRAArm	Dragon, LAWs	0.2	1	0.6
Mortar	81 mm, 60 mm.	0.4	1.2	0.8
Sm Arm	Small Arms	.15	.15	.15
SP Arty	100+ Mortar	1.5	5	4
Td Arty	122+ mm Gn/Hw	1.0	3	2.5
At. Hel	AH-64, Hind	3.5	10	7
Adef	20+ mm SP ADA	0.4	1.5	1.2

Table 4.1. RAND's Firepower Score Values. The weapon categories are not comprehensive. More details can be found in [Ref. 7:p. 88].

Weapon Groups	Personnel	Tank	APC	ARTY	RKTL	ATH	MTR	ATL	FLAME /MG	AA
Firepower Scores	1	100	20	80	90	25	20	20	10	25

Table 4.2. Firepower Score Values Used for the Nine Weapon Groups in KDB. These scores are computed relative to RAND's firepower scores.

2. The Application of the ATLAS Ground Attrition Method to the All Combat Units Data (ACUD)

This section applies the ATLAS ground attrition process to the All Combat Units data (ACUD). The data used in this section counts all combat units, including active, inactive, contact and out of contact units. To aggregate the combat power, the firepower score values presented in Table 4.2 are used. The following section explains the process step by step.

a. Data

The formation of the data from the KOSAVE II report is presented in Chapter III, Section 4. The data, ACUD, used in this section was presented in Chapter III.C.4. OH personnel and weapon strengths for both sides are shown in Tables 3.1 and 3.2 respectively. Personnel and weapon losses of the German and the Soviet forces are presented in Tables 3.3 and 3.4 respectively.

b. Combat Power

The firepower score approach measures the combat power of a unit by summing the combat power values for each weapon group in the unit [Ref. 6:pp. 4-3]. The data used in this study represent only the southern front part of the Battle of Kursk. As presented in Chapter III, all units are aggregated into one level. Most firepower score models represent the unit at the division level, but the currently available data does not represent the unit-level engagements. That is, it is not known which German division engaged which Soviet unit.

The combat power of one side is computed as follows. First, the combat power values for each weapon type are calculated by multiplying the number of weapons by the corresponding firepower score value of that weapon type. Then, the combat power values of the nine weapon groups are summed to represent the total combat power of that side. The combat power is computed each day using personnel on hand and weapon numbers. Equation 1.1 is used to compute the combat power of a force.

Table 4.3 presents the combat power value for each weapon type. Combat power shows the relative combat power of the Germans on each day. The first day has the

highest combat power, or 4 July 1943, the beginning of the battle. Table 4.4 shows the combat power values for each weapon type and the combat power of the Soviets. The Germans are outnumbered at least 2:1 in tanks and at least 3:1 in heavy antitank (ATH), machineguns (Flame/MG), mortars, and light antitank (ATL) weapons throughout the battle. German superiority exists only for artillery, rocket launchers, and armored personnel carriers (APC). In fact, the German artillery combat power value is greater than their tank combat power. The German artillery combat power does not change much during the campaign as artillery suffers relatively few casualties when compared to tanks or APCs. The major fluctuation is seen in steadily declining tank combat power values. On the other hand, all Soviet weapon combat powers drop consistently throughout the battle.

c. Computation of Force Ratio

Once the combat power of both sides is computed, the next step is to calculate the force ratio. Force ratio is always determined by dividing the combat power of all attacking forces by the combat power of all defenders forces.

The force ratio represents the relative combat power in the battle. Table 4.5 shows the combat powers for the Germans and Soviets and the attacker's force ratio. The attacking side and defender's postures are also presented in this table. The process as to how the attacking side is determined is explained in Chapter III Section C.3. This force ratio does not represent other qualitative combat variables, such as leadership, fatigue, morale, surprise, terrain and training. These variables, which might have a significant

effect on the combat, are not explicitly represented in the ATLAS ground attrition process.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power (in millions)
1	301341	98600	**100320**	23980	30600	15450	29780	9760	19380	26200	**0.655**
2	297205	74900	**99920**	23700	30420	14750	29660	9660	19310	26125	**0.625**
3	293960	67300	**98960**	23160	30420	13725	29580	9620	19170	25550	**0.611**
4	306659	59600	**103680**	22820	30420	14650	32460	10500	20630	26275	**0.627**
5	303879	49000	**103680**	22580	30420	14750	31920	10360	20570	26275	**0.613**
6	302014	54800	**102480**	23440	30150	14875	31740	10280	20550	26175	**0.616**
7	300050	56300	**102880**	23220	30150	14600	31700	10260	20270	26450	**0.615**
8	298710	50000	**101440**	23140	30060	14300	31580	10120	20190	26125	**0.605**
9	299369	49500	**100960**	23100	29970	13700	31820	9980	20120	26425	**0.604**
10	297395	48000	**100640**	23040	29970	13675	31680	9740	20090	26075	**0.600**
11	296237	42600	**100640**	22960	29790	13750	31520	9620	19950	25975	**0.593**
12	296426	49500	**100400**	23020	29700	14000	31500	9400	19890	26175	**0.600**
13	296350	55700	**100640**	23080	29700	13950	31480	9420	19880	26250	**0.606**
14	295750	58800	**100000**	23120	29070	14025	31460	9300	19780	26475	**0.607**

Table 4.3. Daily German Combat Power Values for Personnel and Weapon Type. The last column shows the aggregate combat power of the Germans on each day. Notice that the artillery combat power values do not change much.

Day	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power(in millions)
1	507698	239600	56400	10140	28350	56075	143260	328140	64080	17600	**1.451**
2	498884	236700	54080	10020	28080	54175	137660	314540	61190	17800	**1.413**
3	489175	206400	52880	9800	27720	51150	132660	301540	57870	17000	**1.346**
4	481947	175400	51840	9540	27810	48075	129160	290740	55230	16425	**1.286**
5	470762	149500	51200	9160	27720	45925	124880	281020	52740	16300	**1.229**
6	460808	140600	50320	9260	27630	44375	119720	268980	50270	16250	**1.188**
7	453126	135100	50240	9240	27360	42450	115640	259280	48500	16200	**1.157**
8	433813	97700	49040	8640	27180	40700	109960	244900	45830	16100	**1.073**
9	423351	97800	48480	8480	26820	39775	107120	237020	44340	16100	**1.049**
10	415254	90700	48240	8360	26550	39675	104040	230840	42300	16075	**1.022**
11	419374	88300	48080	8340	26550	38475	101820	226020	41340	16075	**1.014**
12	416666	98500	48000	8340	26370	39275	98320	221860	39970	16100	**1.013**
13	415461	97800	48160	8340	26190	39350	98660	221580	39720	16125	**1.011**
14	413298	94800	47280	8180	25920	38550	98040	219340	38500	16125	**1.000**

Table 4.4. Daily Soviet Combat Power Values for Personnel and Weapon Type. The last column shows the aggregate combat power of the Soviets on each day. The tank combat power values decrease dramatically during the campaign.

Day	German Combat Power	Soviet Combat Power	Attacker	Defender's Combat Posture	Attackers Force Ratio
1	655411	1451343	German	Prepared	**0.451**
2	625650	1413129	German	Prepared	**0.442**
3	611445	1346195	German	Prepared	**0.45**
4	627694	1286167	German	Prepared	**0.488**
5	613434	1229207	German	Hasty	**0.499**
6	616504	1188213	German	Hasty	**0.518**
7	615880	1157136	German	Hasty	**0.532**
8	605665	1073863	Soviet	Prepared	**1.773**
9	604944	1049286	Soviet	Prepared	**1.734**
10	600305	1022034	German	Hasty	**0.587**
11	593042	1014374	German	Hasty	**0.584**
12	600011	1013401	German	Hasty	**0.592**
13	606450	1011386	Soviet	Prepared	**1.667**
14	607780	1000033	Soviet	Prepared	**1.645**

Table 4.5. This Table Presents the Combat Power of Both Forces, Attacking Side, Defenders Combat Posture, and Attackers Force Ratio. Notice that for each day the Soviet combat power is significantly greater than the German combat power.

d. Casualty Rates

Casualty rates for the day's battle are computed from the ATLAS attrition equations [Ref. 13] for the attacker and defender separately. These equations are:

$$\text{Attackers casualty rate} = a * (x/y)^{-\alpha} + \beta \tag{4.1}$$

$$\text{Defenders casualty rate} = b * (y/x)^{-e} + \alpha \tag{4.2}$$

Where;

x = attackers combat power,

y = defenders combat power,

a, b, e, α, and β depend on the combat posture. These variables are taken from notes [Ref. 13] provided by Dr. James Taylor and presented in Table 4.6.

Engagement Type	A	b	α	β	d	E
Fortified	0.03593	0.012	0.008	0.02407	0.9	0.899021312
Prepared	0.02781	0.012	0.008	0.01919	0.87	0.899021312
Hasty	0.022384	0.015	0.01	0.016616	0.95	0.929916219
Meeting	0.02	0.02	0.011	0.011	0.89	0.89166815
Withdrawal	0.015434	0.0085	0.006	0.004566	0.93	0.879364086
Delay	0.015434	0.006	0.004	0.004566	0.93	0.778385397
Disorganized Retirement	0.015434	0.012	0.008	0.004566	0.93	0.899021312
Rout	0.015434	0.015	0.01	0.004566	0.93	0.929916219
Holding	-	-	0.009	0.009	0	0

Table 4.6. The Variables Used in Equations 4.1 and 4.2 to Estimate Casualties.

Day	Attacker	Attackers Casualty Rate	Defenders Casualty Rate	Germans Casualty Rate	Soviets Casualty Rate
1	German	0.0472	0.0139	**0.0472**	**0.0139**
2	German	0.0472	0.0138	**0.0472**	**0.0138**
3	German	0.0472	0.0139	**0.0472**	**0.0139**
4	German	0.0472	0.0143	**0.0472**	**0.0143**
5	German	0.0392	0.0179	**0.0392**	**0.0179**
6	German	0.0391	0.0181	**0.0391**	**0.0181**
7	German	0.0391	0.0183	**0.0391**	**0.0183**
8	Soviet	0.0469	0.0281	**0.0281**	**0.0469**
9	Soviet	0.0469	0.0277	**0.0277**	**0.0469**
10	German	0.0391	0.0191	**0.0391**	**0.0191**
11	German	0.0391	0.0191	**0.0391**	**0.0191**
12	German	0.0391	0.0192	**0.0391**	**0.0192**
13	Soviet	0.0469	0.0270	**0.0270**	**0.0469**
14	Soviet	0.0469	0.0268	**0.0268**	**0.0469**

Table 4.7. Estimated Attacker Combat Power Casualty Rates are Always Higher than the Defenders. For the first five days casualty rates are very close for both sides. The Soviets have nearly the same estimated casualty rate for the days they attack. Likewise, the Germans have consistent estimated casualty rates on the days they defend.

Table 4.7 presents the estimated combat power casualty rates for the attacker and the defender. The attacker always has a higher casualty rate than the defenders. For the first five days of the battle, combat power casualty rates remain fairly constant for both sides.

The only variables that might be changed are the combat posture and the force ratio in the equations. The force ratios in this model are very close to each other except for the days on which the Soviets attack, since there are only two different engagement types. These facts cause the estimated casualty rates not to vary very much.

After computing the combat casualty rates, the issue is how to allocate them to the different weapon groups, or in other words, how to define the personnel casualties, APC, artillery and tank losses. This process is addressed in the following section.

e. Distribution of Combat Power Casualty Rates

One of the problems with traditional force ratio models is that the loss rates in each category of weapon are the same as the combat power casualty rate. Without KV scoreboard, the ATLAS algorithm has no direct way to allocate casualties to the different systems. Historical facts or the results from the higher-resolution combat models tell us that different systems suffer different casualty rates.

From the historical facts, it is clear that the loss rates of tanks tend to be higher than the artillery [Ref. 3]. In the ATLAS attrition structure, a process for the allocation of combat casualty rate to weapon categories is not implemented. In order to determine the weapon loss rates, we use linear regression analysis. We choosed the regression analysis as the best way to distribute casualty rates to the different weapon systems to allow comparisons between ATLAS and other models (SFS and QJM) that explicitly distribute casualties to systems. That is, we are using the actual data to do what the KV scoreboards would do. Consequently, the ATLAS fits artificially well for weapon

losses. Figures 4.1a shows the German personnel loss percentages versus combat power loss percentages. Figures 4.1b shows the Soviet tank loss percentages versus combat power loss percentages.

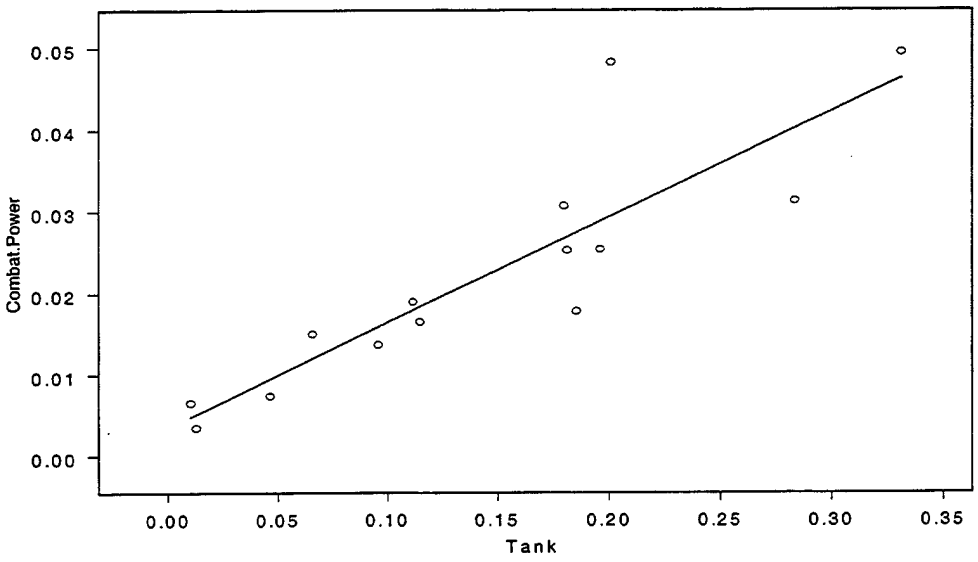

Figure 4.1a. The German Combat Power Casualty Percentages Versus Tank Loss Percentages.

From Figure 4.1a, it can be seen that the pairwise relationship between the German combat power losses and tank losses are positively correlated with a couple outline points. These are the points in which the peak German tank losses occurred. These days are the 2nd and 3rd days of the battle, during which the Germans hit the fortified Soviet defense at the very beginning of the campaign. Figure 4.1b shows that the Soviet combat power losses and personnel losses have a stronger linear trend. The other pairs of weapon system casualties look qualitatively similar to these.

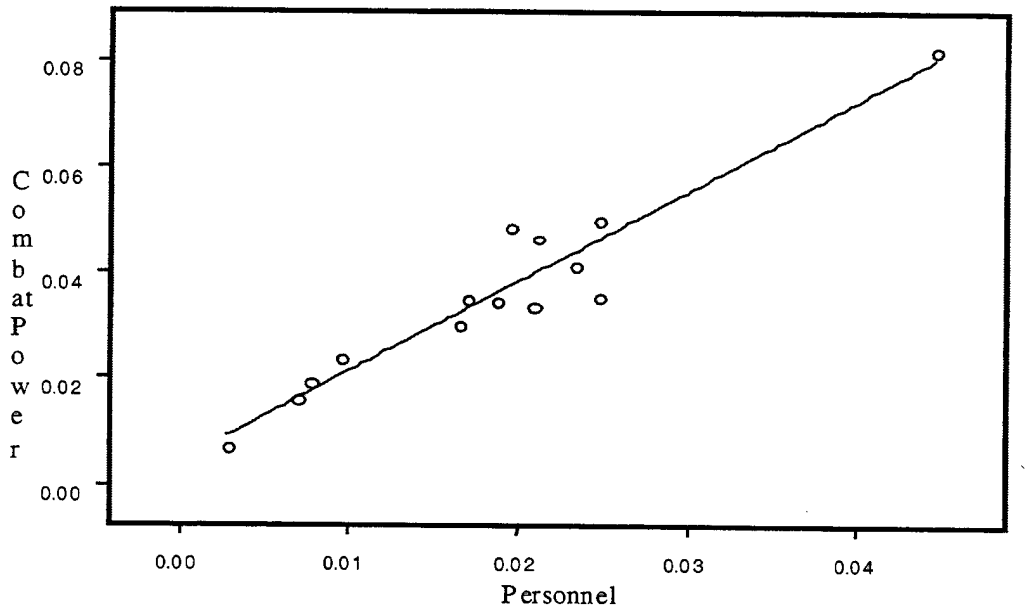

Figure 4.1b. The Soviet Combat Power Casualty Percentages Versus Personnel Loss Percentages.

For 14 days of the battle, we use the personnel casualty percentage as the predictor variable and the weapon loss percentages as the response in fitting a linear regression. This gives the least square estimates of the percent casualties of each weapon categories as a multiplier of personnel casualties. Using the S-Plus software and linear regression without an intercept, coefficient values are calculated for each of the weapon categories. Tables 4.11 and 4.12 present the daily personnel casualties and weapon losses of the Germans and Soviets.

$$Yi = \beta 1 * \text{Percent Personnel Casualties} + \text{error} \qquad (4.3)$$

Where;

Yi = Percent Casualties of type i weapon.

$\beta 1$ = Coefficient to adjust weapon category percent casualties.

The data presented in Tables 3.3 and 3.4 in Chapter III.C.4 is used to fit the linear regression for the German and Soviet sides separately. In the regression, the data for the first day of the battle is dropped. It is clear from the low casualty level that this day is a large outlier. This omission is supported by the historical facts and from the previous studies of Clemens [Ref. 10: p.212], and Turkes [Ref. 11]. The Battle of Kursk did not actually begin in earnest until July 5, 1943. Therefore, the models are fit only to 14 days of the battle. Tables 4.8 and 4.9 present a brief summary of linear regression results.

Categories	**Personnel**	**Tank**	**ARTY**	**APC**	**RKTL**	**ATH**	**MTR**	**ATL**	**Flame/ MG**	**AA**
Coefficients	**1.00**	**16.96**	**0.95**	**1.44**	**0.23**	**4.49**	**1.04**	**1.17**	**0.63**	**1.06**
Pr(>|t|)	-	0.00	0.00	0.00	**0.24**	0.00	5e-4	1e-3	1e-4	9e-4
R-Squared	-	0.83	0.78	0.71	**0.10**	0.88	0.62	0.55	0.71	0.58
F-statistic	-	2e-6	1e-6	6e-5	**0.24**	1e-8	4e-4	1e-3	5e-5	8e-4

Table 4.8. German Regression Results for All Combat Unit Data. Only the RKTL weapons result is not significant.

Categories	**Personnel**	**Tank**	**ARTY**	**APC**	**RKTL**	**ATH**	**MTR**	**ATL**	**Flame/ MG**	**AA**
Coefficients	**1.00**	**7.09**	**0.81**	**1.11**	**0.38**	**1.82**	**1.52**	**1.66**	**1.93**	**0.34**
Pr(>|t|)	-	0.00	0.00	0.00	5e-4	0.00	0.00	0.00	0.00	**0.04**
R-Squared	-	0.89	0.77	0.82	0.61	0.90	0.90	0.94	0.92	**0.27**
F-statistic	-	1e-7	1e-5	3e-6	5e-4	1e-6	5e-8	2e-9	2e-8	**0.04**

Table 4.9. Soviet Regression Results for Contact Combat Unit Data. The results are not significant only for AA weapons.

Regression results show that on average the Germans will lose 16.96% of tanks, 0.95% of artillery, 1.44% of APC, 0.23% of rocket launchers, 4.49% of heavy antitank weapons, 1.04% of mortar, 1.17% of light anti tank weapons, 0.63% of heavy machine guns and 1.06% of air defense weapons for every 1% of personnel losses.

Likewise, the Soviets suffered 7.09% of tanks, 0.81% of artillery, 1.11% of APC, 0.38% of rocket launchers, 1.82% of heavy anti tank weapons, 1.52% of mortar, 1.66% of light anti tank weapons, 1.93% of heavy machine guns and 0.34% of air defense weapon losses for every 1% of personnel losses.

When the coefficients of the Germans and the Soviets are compared, it is seen that they are not equal. The big difference is observed in tank losses. While the Germans are losing 16.96% of their tanks for their 1% personnel losses, the Soviets will suffer only 7.09% of their tanks for every 1% of their personnel losses. Also, the ATH, Flame/MG, and AA coefficients are quite different. Since the two sides had different mixes of weapons and attacked in different proportions we would not expect the coefficients to be the same, however we can formally test this. In order to test if the losses could be the same, i.e., the differences are explainable by simple random variation, a regression analysis is done. The procedure used follows.

First, a new data set was built which combines the German and Soviet loss percentages by weapon system (nine weapon categories). Now, the new data contains 28 points, 14 days from the Soviets and 14 days from the Germans. The regression formula is constructed as:

$$Y_i = \beta_1 * X_1 + \beta_2 * IG * X_1 \qquad (4.4)$$

Where;

Yi = Percent Casualties of type i weapon

X1 = Percent Personnel casualties

IG = Indicator variable. It is 1 for the Germans and zero for the Soviets

In order to test whether or not the Germans' and the Soviets' coefficients are the same, a hypothesis test is done. The null hypothesis is H_0: β2 $= 0$, and the alternative hypothesis is, H_a: β2 $\neq 0$. The regression results from the S-Plus software are presented in Table 4.10. The p-values of the coefficients from the results are used to test our hypothesis. The p-values are the probability we would see data as or more extreme if H_0 is true.

	Categories	**Tank**	**ARTY**	**APC**	**RKTL**	**ATH**	**MTR**	**ATL**	**Flame/MG**	**AA**
Personnel	Coefficient	7.09	0.81	1.1	0.38	1.82	1.51	1.66	1.93	0.34
	Pr(>|t|)	0.0	0.0	0.0	1e-4	0.0	0.0	0.0	0.0	0.016
Personnel:IG	Coefficient	9.9	0.14	0.32	-0.14	2.67	-0.46	-0.49	-1.30	0.71
	Pr(>|t|)	0.0	**0.55**	**0.31**	**0.5**	0.0	**0.13**	**0.12**	2e-4	0.043
R2	-	0.86	0.77	0.79	0.45	0.87	0.87	0.89	0.91	0.41
F-statistic	-	0.0	0.0	0.0	3e-4	0.0	0.0	0.0	0.0	9e-4

Table 4.10. The Regression Results for the Hypothesis that the German and Soviet Losses are the Same for the ACUD Data Set.

Our primary concern for the regression outputs is the p-values for the indicator variable-Personnel:IG. The p-values for the tank, heavy antitank (ATH), and Flame/MG weapon groups strongly suggest that H_0 can be rejected at the significant level

of 0.05. Thus, the German and the Soviet losses for these weapon groups are not the same.

The p-values for the artillery, APC, RKTL, mortar, and ATL weapon groups give support that H_0 cannot be rejected. Thus, the German and Soviet losses for these weapon groups may be the same. These results are for the ACUD data set. The results are compared with the ones for the other data sets.

The casualty percentage relation between weapon categories and personnel strength was determined. Now, the problem is using these ratios to distribute combat casualty rates to weapon groups. Let,

X be personnel loss percentage,

R_i the regression coefficient of i type weapon category ($i = 1, 2...n$), and

CCR be combat casualty rate, then

$$\sum_{i-1}^{n} Ri * X = n * CCR \qquad (4.5)$$

Using the ATLAS curves and Equation 4.5 above, the personnel casualty rate (X) is solved. Then, each weapon category loss rate is computed by multiplying the personnel casualty rate (X) with their regression coefficient (R). This distributes the casualties according to the regression while ensuring the CCR is as estimated by the ATLAS curves. Personnel casualty rates and weapon loss rates are presented in Tables 4.11 and 4.12 for both forces. Note: this will provide a more accurate fit than the

Firepower score algorithms would do in practice because they are tuned to the actual results, rather than estimated by an algorithm.

Day	Combat Power	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
1	**4.72**	1.63	**27.59**	1.55	2.34	0.39	7.31	1.71	1.91	1.03	1.73
2	**4.72**	1.63	**27.59**	1.55	2.34	0.39	7.31	1.71	1.91	1.03	1.73
3	**4.72**	1.63	**27.59**	1.55	2.34	0.39	7.31	1.71	1.91	1.03	1.73
4	**4.72**	1.63	**27.58**	1.55	2.34	0.39	7.31	1.71	1.91	1.03	1.73
5	**3.92**	1.35	**22.90**	1.29	1.94	0.32	6.07	1.42	1.58	0.85	1.43
6	**3.91**	1.35	**22.89**	1.29	1.94	0.32	6.07	1.42	1.58	0.85	1.43
7	**3.91**	1.35	**22.89**	1.29	1.94	0.32	6.07	1.42	1.58	0.85	1.43
8	**2.81**	0.97	**16.42**	0.93	1.39	0.23	4.35	1.02	1.14	0.61	1.03
9	**2.77**	0.95	**16.19**	0.91	1.37	0.23	4.29	1.00	1.12	0.60	1.01
10	**3.91**	1.35	**22.88**	1.29	1.94	0.32	6.06	1.41	1.58	0.85	1.43
11	**3.91**	1.35	**22.88**	1.29	1.94	0.32	6.06	1.41	1.58	0.85	1.43
12	**3.91**	1.35	**22.88**	1.29	1.94	0.32	6.06	1.41	1.58	0.85	1.43
13	**2.70**	0.93	**15.79**	0.89	1.34	0.22	4.19	0.98	1.09	0.59	0.99
14	**2.68**	0.92	**15.66**	0.88	1.33	0.22	4.15	0.97	1.08	0.58	0.98

Table 4.11. Estimated Daily German Loss Percentages. Due to rounding off, some days seem to have the same rate. Note the high estimated tank losses.

f. Results

In this section, actual and estimated losses are compared for daily results. The estimated combat power losses are done by ATLAS curves, and weapon loss rates are calculated by ATLAS and our regression analysis. The quality of the fits is presented in the comparison figures. The figures are a good way to show the pattern of the fits. For the purpose of comparing different models throughout this thesis, R^2 values are computed, where R^2 is given as:

$$R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{\sum_i (Y - \hat{Y})^2}{\sum_i (Y - \bar{Y})^2} \tag{4.6}$$

where $\hat{Y}$, Y and $\overline{Y}$ denote the estimated value, the real value and the mean value of the Y parameter (daily casualties), which are indexed by days. A greater R^2 value indicates a better fit. It is possible to get a negative R^2 value, implying that the fitted model yields worse results than using the average daily losses as an estimate [Ref. 11: p.52].

Day	Combat Power	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
1	**1.39**	0.78	5.56	0.64	0.87	0.30	1.43	1.19	1.31	1.52	0.27
2	**1.38**	0.78	5.52	0.63	0.87	0.30	1.42	1.18	1.30	1.51	0.27
3	**1.39**	0.79	5.57	0.64	0.88	0.30	1.43	1.19	1.31	1.52	0.27
4	**1.43**	0.81	5.73	0.65	0.90	0.31	1.47	1.23	1.35	1.56	0.28
5	**1.79**	1.01	7.16	0.82	1.13	0.38	1.84	1.53	1.68	1.95	0.35
6	**1.81**	1.03	7.28	0.83	1.14	0.39	1.87	1.56	1.71	1.99	0.36
7	**1.83**	1.04	7.35	0.84	1.16	0.39	1.89	1.58	1.73	2.01	0.36
8	**4.69**	2.65	**18.79**	2.15	2.95	1.01	4.83	4.02	4.42	5.13	0.92
9	**4.69**	2.65	**18.79**	2.15	2.95	1.01	4.83	4.03	4.42	5.13	0.92
10	**1.91**	1.08	7.68	0.88	1.21	0.41	1.97	1.64	1.80	2.09	0.38
11	**1.91**	1.08	7.66	0.87	1.20	0.41	1.97	1.64	1.80	2.09	0.38
12	**1.92**	1.09	7.70	0.88	1.21	0.41	1.98	1.65	1.81	2.10	0.38
13	**4.69**	2.65	**18.80**	2.15	2.95	1.01	4.83	4.03	4.42	5.13	0.92
14	**4.69**	2.65	**18.80**	2.15	2.95	1.01	4.83	4.03	4.42	5.13	0.92

Table 4.12. Estimated Daily Soviet Loss Percentages. Due to rounding off, some days seem to have the same rate. The Soviets have higher estimated casualty rates when they attack.

Table 4.13 presents the R^2 values of the German and Soviet forces. Most of the values are negative, but they are very close to zero. One poor day fit can cause a negative R^2 value. It is better to use the figures and the R^2 values together. Figures give a better insight into understanding the pattern of the fits. Throughout this thesis, only combat power, personnel, and tank figures are given in the chapters.

As mentioned above, one concern with the R^2 values is that they are sensitive to the outliers. For instance, if the model fits poor for one or two days of the

battle, it can give a small R^2 value. Thus, in order to test that the model under/overestimates the battle, a hypothesis test is done. Since the data consists of pairs, actual and estimated values, our hypothesis is whether or not the difference between the actual and estimated values is zero. The null hypothesis will be, H_0: $\mu_D = 0$, and the alternative hypothesis will be, H_a: $\mu_D \neq 0$. Here D is the difference between actual and estimated values. Using S-Plus software, a non-parametric Wilcoxon signed-rank test for all the German and Soviet weapon groups is done. Table 4.14 shows the p-values for this test. Those p-values greater than 0.05 suggest that the null hypothesis cannot be rejected at the significant level of 0.05.

Weapon groups	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
German	-0.80	**0.16**	-0.63	-0.15	-0.11	-0.73	-0.43	-0.79	-0.41	-0.32	-0.01
Soviet	-0.65	-0.19	-0.75	**0.04**	-0.91	-1.31	-1.72	-1.32	-1.69	-0.23	-0.71

Table 4.13. R^2 Values of Personnel Casualties and Weapon Losses. Tank R^2 value for the Germans indicates a better fit.

Weapon groups	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
German	9e-4	0.005	0.02	0.02	**0.54**	0.005	0.01	**0.36**	0.02	**0.14**	0.001
Soviet	**0.21**	**0.32**	**0.32**	**0.85**	**0.05**	**0.11**	**0.09**	**0.09**	0.04	0.46	**0.11**

Table 4.14. The p-Values from the Wilcoxon Signed-Rank Test for ACUD Data Set. The non-significant values are highlighted.

There are only two positive R^2 values: the German tank and the Soviet APC losses. The p-value for the Soviet APC losses supports the R^2 value and the model fits this weapon group. Despite its positive R^2 value, the p-value of the German tank

losses is small. The p-values for the Soviet losses show that the difference between the actual and the estimated losses equal zero except for the Flame/MG and AA groups. For the Germans, the difference between actual and estimated losses of the RKTL, ATL, and AA weapon groups are zero at the significant level of 0.05.

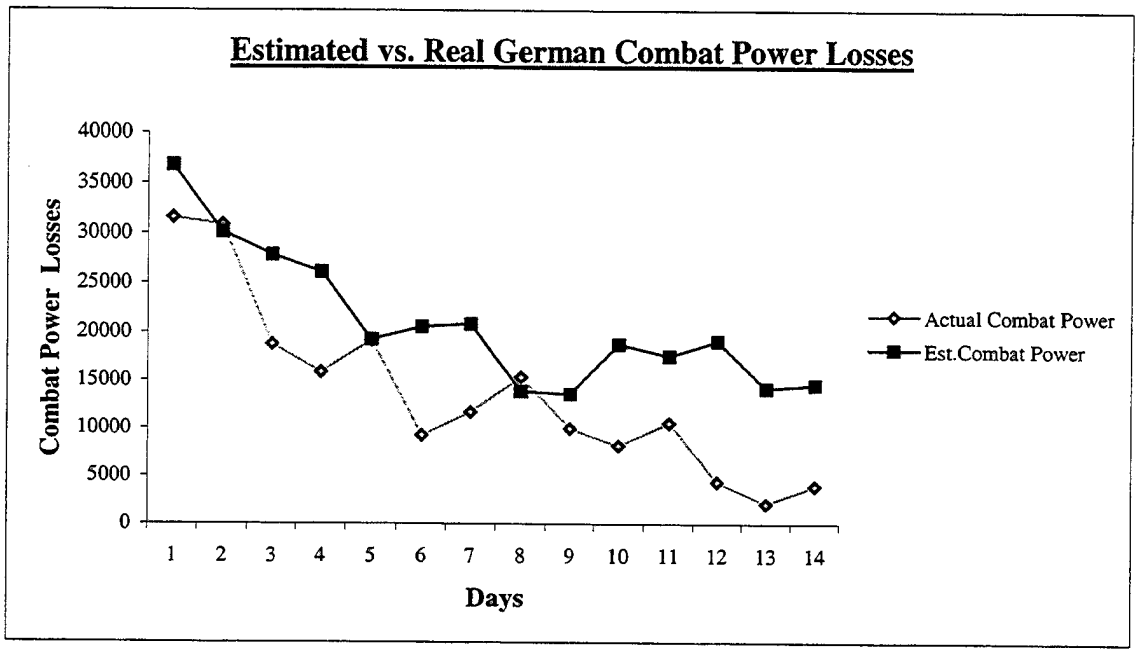

Figure 4.2. Fitted Versus Actual for German Combat Power Losses Applying the ATLAS Attrition Process to All Combat Unit Data. The model overestimates casualties on most days of the battle.

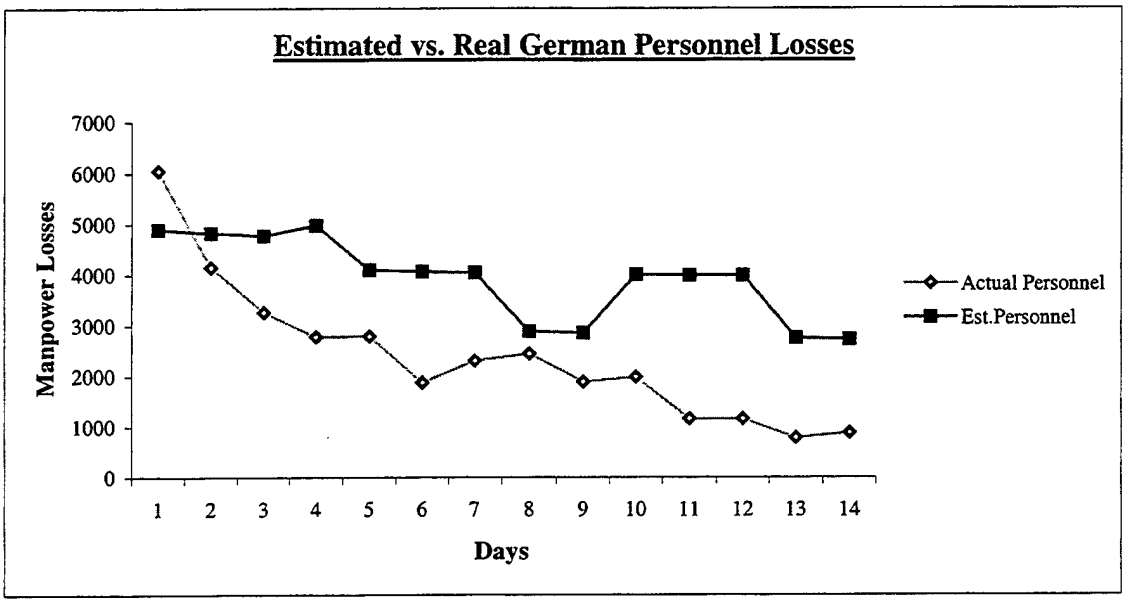

Figure 4.3. Fitted Versus Actual for German Personnel Losses. The model overestimates battle casualties except for the first day.

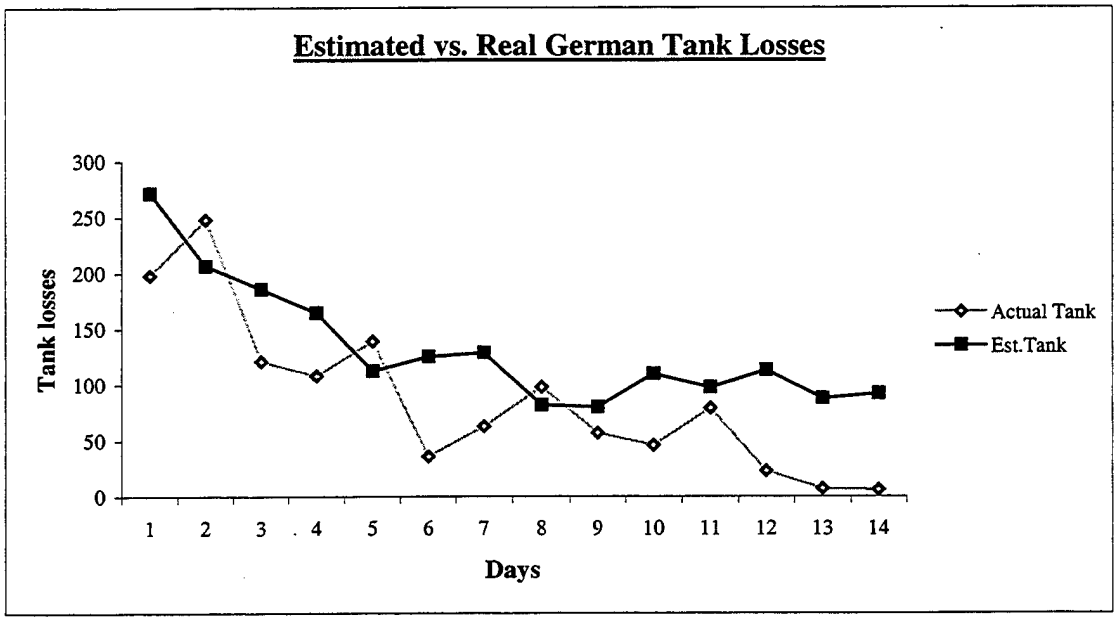

Figure 4.4. Fitted Versus Actual Tank Losses. The trend of the model is very plausible. There is no significant outlier.

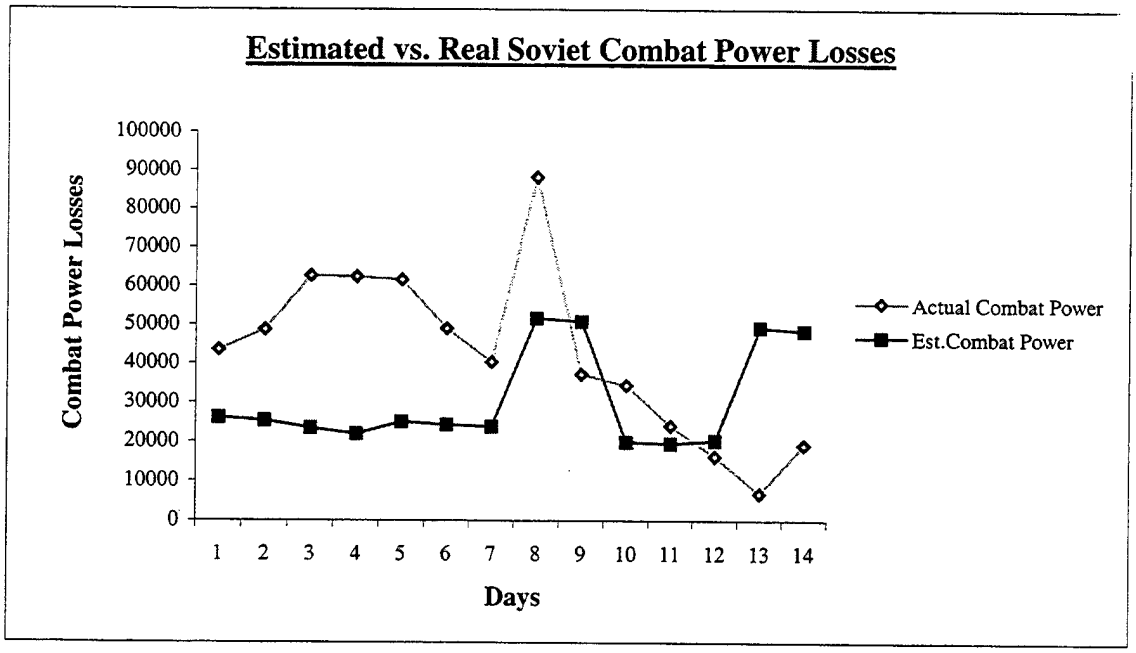

Figure 4.5. Fitted Versus Actual for Soviet Combat Power Losses. The peak four points are the days when the Soviets attack. The model mostly underestimates the battle, with the last two days being a noticeable exception.

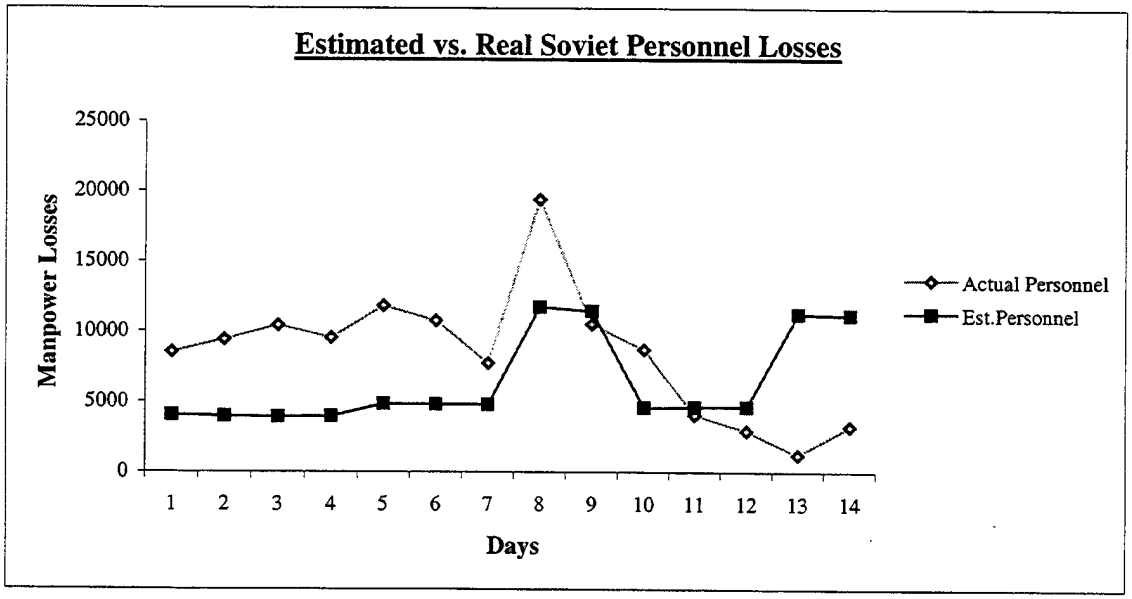

Figure 4.6. Fitted Versus Actual Soviet Personnel Losses. Except the last three days, the model underestimates the battle. Day 8 is the bloodiest day of the battle.

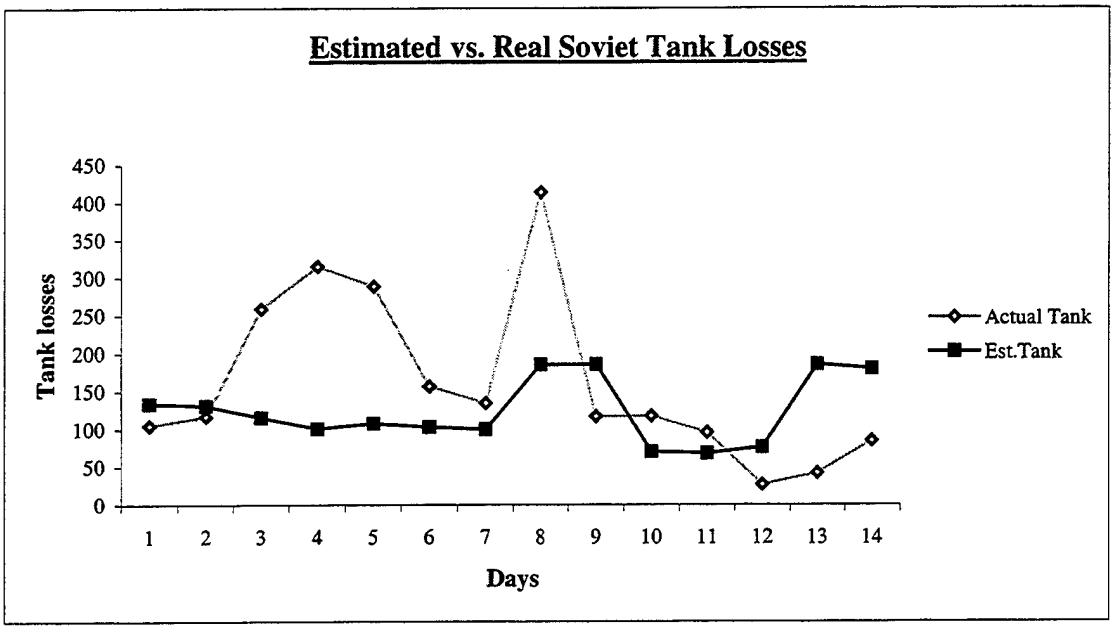

Figure 4.7. Fitted Versus Actual Soviet Tank Losses. The model underestimates battle casualties except for the last three days. Again, day 8 is the heaviest tank battle in history.

Figures 4.2, 4.3, and 4.4 show that ATLAS curves overestimate the losses for the German forces. It is also true that the model overestimates the other weapon losses not presented here. This fact is also supported by the signed rank test values presented in Table 4.14. The resulting patterns in the figures are plausible for German forces. It is clear that the Germans attack with a force ratio of less than one. In such cases, the attacker is expected to take many more casualties than the defender. This might explain the overestimation in the figures. According to the traditional "3:1" rule, the Germans should not have attacked, but they did and the Soviets suffered more casualties. One explanation might be the relative combat effectiveness. On the Eastern Fronts of the two World Wars, the German relative combat effectiveness superiority over the Russians was ranging between the factors of 2.0 and 3.0. [Ref. 3:p. 43]. The Germans have a

higher relative combat effectiveness value, because they were well organized and more professional than their opponents were [Ref. 3:p. 43]. In the ATLAS attrition process, relative combat effectiveness is not considered explicitly.

Unlike the Germans, the Soviet's estimated losses were less than their actual losses. Since they were mostly in defensive positions with a higher force ratio, it was estimated that they would take fewer casualties. However, the fact is that they suffered more losses. The only days in which Soviet casualties were overestimated were the last three days. This is consistent with the fact that after day 9, the battle lost its intensity. The ATLAS equations do not implement the battle intensity as a variable in the attrition process. Thus, roughly the same percentage of casualties on day 8 and 14 were estimated.

Other figures for all weapon groups for both sides can be obtained from the author or his advisor.

3. The Application of the ATLAS Ground Attrition Method to Contact Combat Units Data (CCUD)

The data used in Section 3 includes all unit data of those engaged or not engaged directly in the combat. To investigate the influence of unit activities to attrition, the ATLAS equations are applied to the contact and fighting unit data sets. Since the Germans had a higher percentage in contact than the Soviets, this may help in adjusting the over/underestimation of the models. This section will apply the ATLAS ground attrition process to the contact combat units data (CCUD) set.

a. Data

The formation of the data from the KOSAVE II report is presented in Chapter III, Section 4. The data, CCUD, used in this section was presented in Chapter III.C.5. OH personnel and weapon strengths for both sides are shown in Tables 3.5 and 3.6 respectively. Personnel and weapon losses of the German and Soviet forces are presented in Tables 3.7 and 3.8 respectively.

b. Combat Power and Force Ratio

The procedure, which is used in Chapter IV Section 2.b, is applied to compute the combat power of the forces. The contact unit data is aggregated using the same firepower score values, i.e., see Table 4.2. The new combat powers of the Soviets and Germans, and the attacker's force ratio, are presented in Table 4.15. The same combat postures are used in these calculations.

Day	German Combat Power	Soviet Combat Power	Attacker	Defender's Combat Posture	Attacker Force Ratio
1	590950	478989	German	Prepared	**1.234**
2	595298	601831	German	Prepared	**0.989**
3	582935	616078	German	Prepared	**0.946**
4	579456	633302	German	Prepared	**0.915**
5	567306	685975	German	Hasty	**0.827**
6	543628	728815	German	Hasty	**0.746**
7	585547	895732	German	Hasty	**0.654**
8	579236	843193	Soviet	Prepared	**1.456**
9	580117	840019	Soviet	Prepared	**1.448**
10	573579	809195	German	Hasty	**0.709**
11	582216	755686	German	Hasty	**0.770**
12	584947	684744	German	Hasty	**1.171**
13	507851	661533	Soviet	Prepared	**1.303**
14	509223	687112	Soviet	Prepared	**0.741**

Table 4.15. Estimated Daily German and Soviet Combat Power. Notice that the range of force ratio is [0.63-1.45].

Comparing the contact units force ratio with the all units' force ratio shows that the German force ratio increases while the Soviets force ratio decreases. This is an expected result since the Germans had a higher percentage of units in contact for most of the battle. It gives a higher force ratio for the Germans when they attack and lower force ratio for the Soviets when they attack. The next sections will show how the change in force ratio affects the quality of the fits.

c. Casualty Rates and its Distribution

ATLAS ground attrition equations, which are presented in Section 2.d as Equations 4.1 and 4.2, are used to compute the estimated daily casualty rates. The same combat postures are used in the equations as before. Table 4.16 shows the attackers, defenders, the Germans and the Soviets daily combat casualty rates as estimated by the ATLAS curves. Again, the attacker always has a higher casualty rate than the defender. The Germans have greater estimated casualties during the first 4 days. The Soviets have lower estimated casualties than the Germans except on the days on which they attack. They have almost the same estimated casualty rates on the days they attack.

When the all unit and the contact unit estimated casualty rates are compared, it is observed that estimated casualty rates of contact units data are lower than the all unit data, except for the last two days for the Soviets. This can be explained by the decrease in the Soviet combat forces, since they have a lower engagement percentage when compared with the Germans. However, the pattern of the estimated casualties looks similar. After having computed estimated casualty rates for both forces, the next issue is to allocate them to different weapon categories.

Day	Attacker	Attackers Casualty Rate	Defenders Casualty Rate	Germans Casualty Rate	Soviets Casualty Rate
1	German	0.047	0.022	0.047	0.022
2	German	0.047	0.020	0.047	0.020
3	German	0.047	0.019	0.047	0.019
4	German	0.047	0.019	0.047	0.019
5	German	0.039	0.023	0.039	0.023
6	German	0.039	0.021	0.039	0.021
7	German	0.039	0.020	0.039	0.020
8	Soviet	0.047	0.025	0.025	0.047
9	Soviet	0.047	0.025	0.025	0.047
10	German	0.039	0.021	0.039	0.021
11	German	0.039	0.022	0.039	0.022
12	German	0.039	0.023	0.039	0.023
13	Soviet	0.047	0.023	0.023	0.047
14	Soviet	0.047	0.024	0.024	0.047

Table 4.16. Attacker Casualty Rates are Always Higher than the Defenders. The Soviets have lower estimated casualty rates than the Germans except on the days they attack.

In order to distribute the estimated combat power casualties to different weapon categories, the algorithm used in Section 4.2.d is also used here. The only difference is, instead of all unit regression coefficients given in Tables 4.8 and 4.9, the ones presented in Tables 4.17 and 4.18 are calculated using the contact units data. These coefficients are almost the same as the ones computed using all unit data. Personnel casualty rate and weapon loss rates are presented in Tables 4.20 and 4.21 for both forces.

Categories	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
Coefficients	**1.00**	**15.02**	**0.85**	**1.38**	**0.23**	**4.23**	**1.06**	**1.11**	**0.59**	**1.05**
Pr(>|t|)	-	0.00	0.00	0.00	**0.23**	0.00	6e-4	1e-3	1e-4	8e-4
R-Squared	-	0.80	0.74	0.73	**0.10**	0.89	0.61	0.55	0.72	0.59
F-statistic	-	5e-6	3e-5	4e-5	**0.23**	1e-7	5e-4	1e-3	6e-5	8e-4

Table 4.17. German Regression Results for Contact Combat Unit Data. The results are not significant only for RKTL weapons.

Categories	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
Coefficients	**1.00**	**6.35**	**1.14**	**0.99**	**0.30**	**1.88**	**1.56**	**1.85**	**2.17**	**0.37**
Pr(>|t|)	-	0.00	0.00	0.00	**2e-2**	0.00	0.00	1e-4	0.00	**0.01**
R-Squared	-	0.83	0.73	0.83	**0.34**	0.83	0.92	0.93	0.91	**0.40**
F-statistic	-	2e-6	5e-5	2e-6	**2e-2**	2e-6	1e-8	7e-5	4e-8	**0.01**

Table 4.18. Soviet Regression Results for Contact Combat Unit Data. The results are not significant only for RKTL and AA weapon classes.

In order to test whether or not the German and the Soviet losses are the same, a similar regression analysis to that presented in Section 4.2.e for the ACUD data set is done. The only difference here is that the CCUD data set is used instead of the ACUD data. Table 4.19 shows the results for the regression output for the CCUD data set. Almost the same results with the ACUD data set occur except for the AA losses. The p-value for the AA losses is greater than 0.05, which shows that the German and the Soviet AA losses are the same for the CCUD data set.

	Categories	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
Personnel	Coefficient	6.35	1.14	0.99	0.30	1.89	1.56	1.86	2.1	0.37
	Pr(>|t|)	0.0	0.0	0.0	3e-3	0.0	0.0	0.0	0.0	0.014
Personnel:IG	Coefficient	8.67	-0.1	0.38	-0.07	2.3	-0.5	-0.74	-1.6	0.68
	Pr(>|t|)	1e-3	**0.82**	**0.26**	**0.83**	1e-3	**0.16**	**0.06**	2e-3	**0.06**
R2	-	0.82	0.72	0.81	0.30	0.85	0.90	0.91	0.91	0.47
F-statistic	-	0.0	0.0	0.0	9e-3	0.0	0.0	0.0	0.0	2e-4

Table 4.19. The Regression Results for the Hypothesis that the German and the Soviet Losses are the Same for the CCUD Data Set.

Day	Combat Power	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
1	**4.70**	1.77	**26.57**	1.50	2.44	0.41	7.48	1.88	1.97	1.06	1.87
2	**4.70**	1.77	**26.59**	1.51	2.44	0.42	7.49	1.89	1.97	1.06	1.87
3	**4.70**	1.77	**26.60**	1.51	2.44	0.42	7.49	1.89	1.97	1.06	1.87
4	**4.70**	1.77	**26.60**	1.51	2.44	0.42	7.49	1.89	1.97	1.06	1.87
5	**3.90**	1.47	**22.09**	1.25	2.03	0.34	6.22	1.57	1.64	0.88	1.55
6	**3.91**	1.47	**22.10**	1.25	2.03	0.34	6.22	1.57	1.64	0.88	1.55
7	**3.91**	1.47	**22.12**	1.25	2.03	0.35	6.23	1.57	1.64	0.88	1.55
8	**2.48**	0.93	**14.04**	0.80	1.29	0.22	3.95	1.00	1.04	0.56	0.99
9	**2.47**	0.93	**14.00**	0.79	1.29	0.22	3.94	0.99	1.04	0.56	0.98
10	**3.91**	1.47	**22.11**	1.25	2.03	0.35	6.23	1.57	1.64	0.88	1.55
11	**3.91**	1.47	**22.10**	1.25	2.03	0.34	6.22	1.57	1.64	0.88	1.55
12	**3.90**	1.47	**22.09**	1.25	2.03	0.34	6.22	1.57	1.64	0.88	1.55
13	**2.32**	0.87	**13.14**	0.74	1.21	0.21	3.70	0.93	0.97	0.52	0.92
14	**2.37**	0.89	**13.41**	0.76	1.23	0.21	3.78	0.95	0.99	0.99	0.94

Table 4.20. Estimated Daily German Loss Percentages. Due to rounding off, some days seem to have the same rate. A Large amount of tank losses is estimated.

Day	Combat Power	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
1	2.25	1.28	8.10	1.46	1.27	0.39	2.40	1.99	2.37	2.77	0.48
2	1.99	1.13	7.16	1.29	1.12	0.34	2.12	1.76	2.10	2.45	0.42
3	1.94	1.10	6.99	1.26	1.10	0.33	2.07	1.72	2.05	2.39	0.41
4	1.91	1.08	6.87	1.23	1.08	0.33	2.04	1.69	2.01	2.35	0.40
5	2.26	1.28	8.13	1.46	1.28	0.39	2.41	2.00	2.38	2.78	0.48
6	2.14	1.21	7.72	1.39	1.21	0.37	2.29	1.89	2.26	2.64	0.45
7	2.01	1.14	7.24	1.30	1.14	0.35	2.15	1.78	2.12	2.48	0.42
8	**4.69**	**2.66**	**16.90**	**3.04**	**2.65**	**0.81**	**5.01**	**4.15**	**4.94**	**5.78**	**0.99**
9	**4.69**	**2.66**	**16.90**	**3.04**	**2.65**	**0.81**	**5.01**	**4.15**	**4.94**	**5.78**	**0.99**
10	2.09	1.18	7.53	1.35	1.18	0.36	2.23	1.85	2.20	2.57	0.44
11	2.18	1.23	7.84	1.41	1.23	0.37	2.32	1.92	2.29	2.68	0.46
12	2.30	1.30	8.27	1.49	1.30	0.39	2.45	2.03	2.42	2.83	0.49
13	**4.69**	**2.66**	**16.91**	**3.04**	**2.65**	**0.81**	**5.01**	**4.15**	**4.95**	**5.78**	**0.99**
14	**4.69**	**2.66**	**16.91**	**3.04**	**2.65**	**0.81**	**5.01**	**4.15**	**4.95**	**5.78**	**0.99**

Table 4.21. Estimated Daily Soviet Loss Percentages. Due to rounding off, some days seem to have the same rate. The Soviets have higher estimated casualty rates when they attack.

d. Results

For the purpose of comparing actual and estimated casualties, R^2 values and figures are used together. Table 4.22 presents the R^2 values for the Germans and Soviets, with an R^2 = 1 being a perfect fit. Positive values for the tank and combat power losses of the Germans occur as well as for the APC losses of the Soviets. Although it is positive, the German combat power R^2 value is very small. The Soviet combat power R^2 value is still negative. Overall, R^2 values for both sides are better than the ones computed in Section 4.2.e using the all combat unit data (ACUD). The results show that the ATLAS ground attrition model and the regression method to distribute losses fit better to the contact combat units than the all combat units data of the Battle of Kursk. The model overestimates the German loses in the ACUD data set, while it underestimates the Soviet losses. This over/underestimation is due to the lower German force ratio in the ACUD data set. On the other hand, in CCUD data set, the Germans have higher force ratios than the ACUD data set due to their higher percentage of units in contact. The higher German force ratio results in lower casualty estimates for the Germans and higher casualty estimates for the Soviets. As a result, the ATLAS curve fits better in the CCUD data set. In the following pages, real versus estimated losses are plotted to give a better understanding of the quality of the fits.

As in Chapter IV.2.f, a Wilcoxon signed-rank test is done to test the hypothesis whether or not the difference between actual and estimated losses are statistically significant. The p-values of the tests are presented in Table 4.23. The

difference between the actual and estimated losses for the German tank, APC, RKTL, ATL, Flame/MG, and AA weapon groups is accepted to be zero at the significant level of 0.05. This result suggests that the model fit better for the CCUD data set than the ACUD data set, which is also supported by the R^2 values. But this is not true for the Soviet losses, since the p-values suggest a better fit for some weapon groups while not for others.

Weapon groups	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
German	-0.86	**0.26**	-0.70	-0.08	-0.13	-0.64	-0.59	-0.80	-0.42	-0.29	**0.08**
Soviet	-0.33	-0.20	-0.45	**0.08**	-0.15	-0.78	-1.12	-0.92	-1.19	-0.26	-0.53

Table 4.22. R^2 Values of Personnel Casualties and Weapon Losses. Tank and combat power values for the Germans and the APC value of the Soviets indicate a better fit.

Weapon groups	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
German	0.004	**0.05**	0.04	**0.05**	**0.1**	0.03	0.03	**0.2**	**0.06**	**0.2**	0.001
Soviet	0.02	0.03	**0.3**	**0.5**	0.03	**0.2**	0.04	0.04	**0.05**	**1**	0.02

Table 4.23 The p-Values from the Wilcoxon Signed-Rank Test for CCUD Data Set. The non-significant values are highlighted.

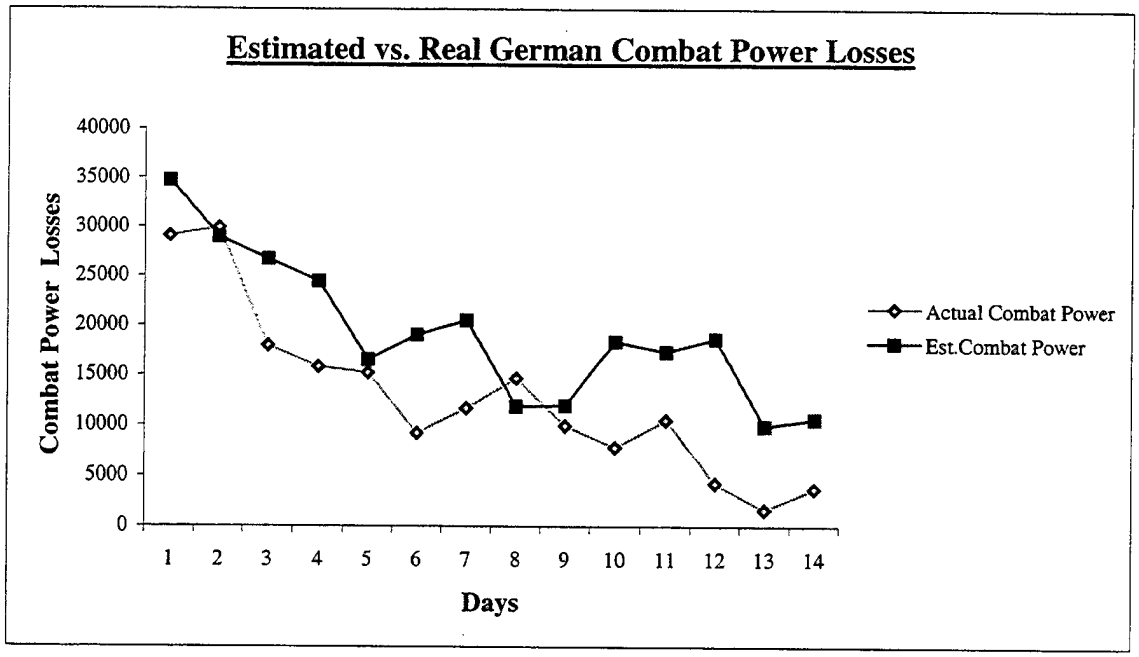

Figure 4.8. Fitted Versus Actual for the German Combat Power Losses Applying ATLAS Attrition Process to Contact Combat Unit Data. The model overestimates on most days except for the second, fifth and eighth days

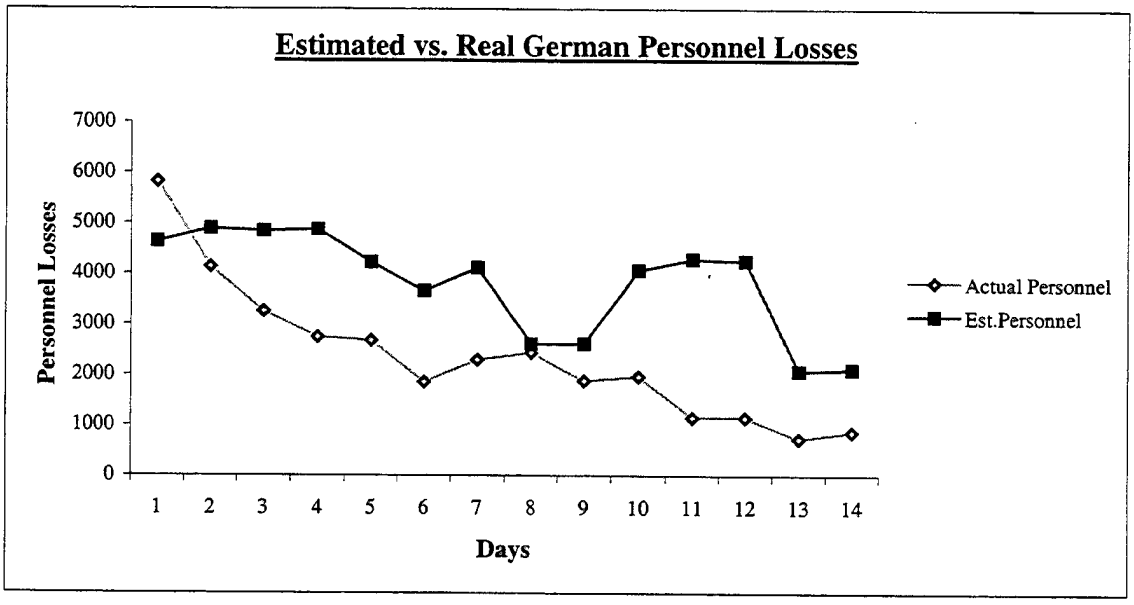

Figure 4.9. Fitted Versus Actual for the German Personnel Losses. The model overestimates battle casualties except for the first and eighth day.

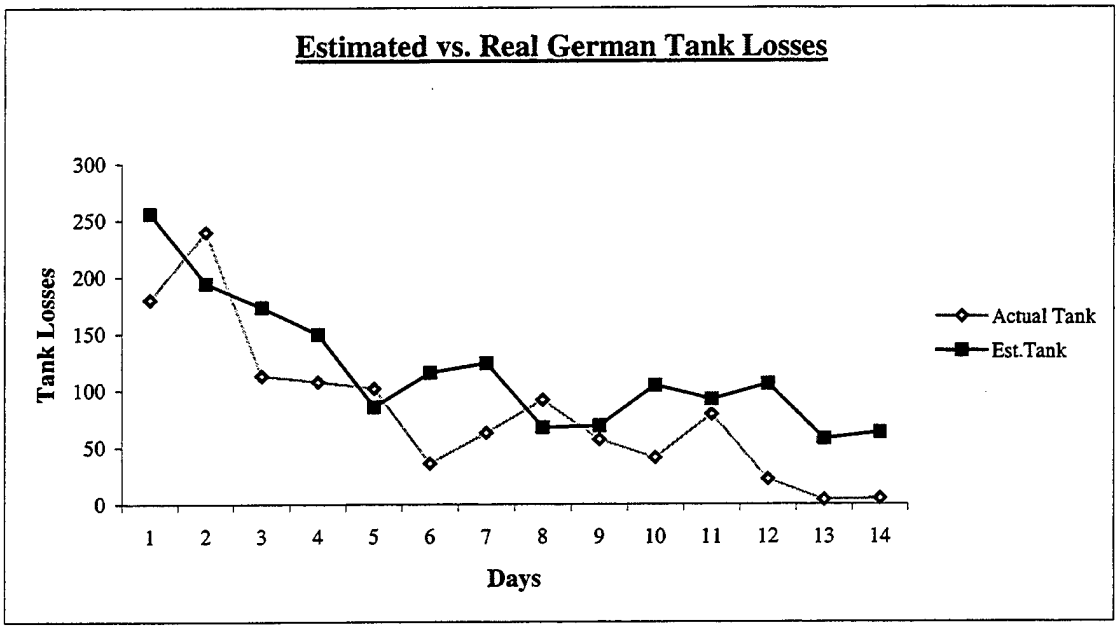

Figure 4.10. Fitted Versus Actual Tank Losses. The model catches the battle trend, but overestimates battle casualties towards the end. There is no significant outlier.

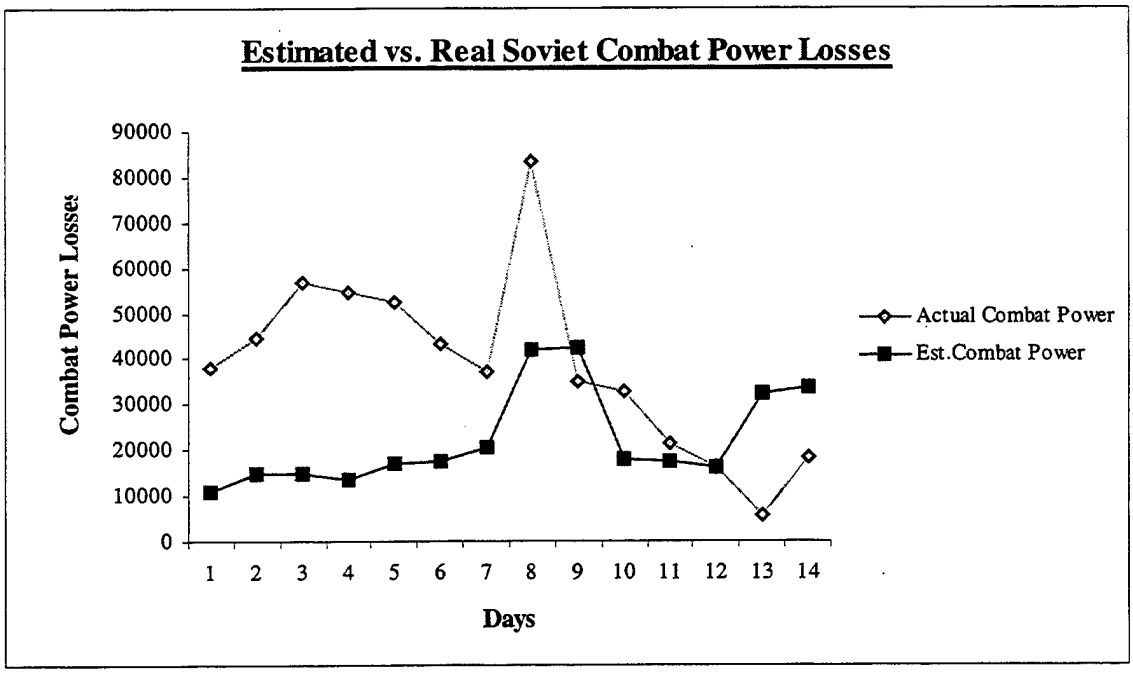

Figure 4.11. Fitted Versus Real for Soviet Combat Power Losses. The peak four points are the days when the Soviets attack. The model mostly underestimates battle casualties.

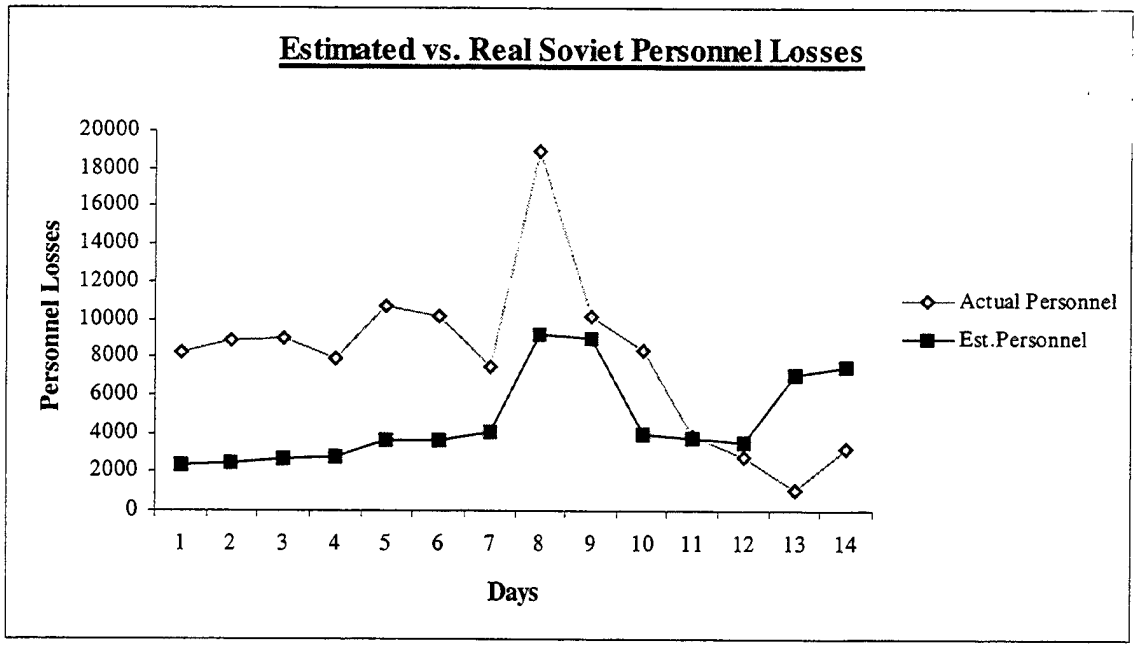

Figure 4.12. Fitted Versus Real the Soviet Personnel Losses. Except for the last two days, the model underestimates battle casualties. Day 8 is the bloodiest day of the battle.

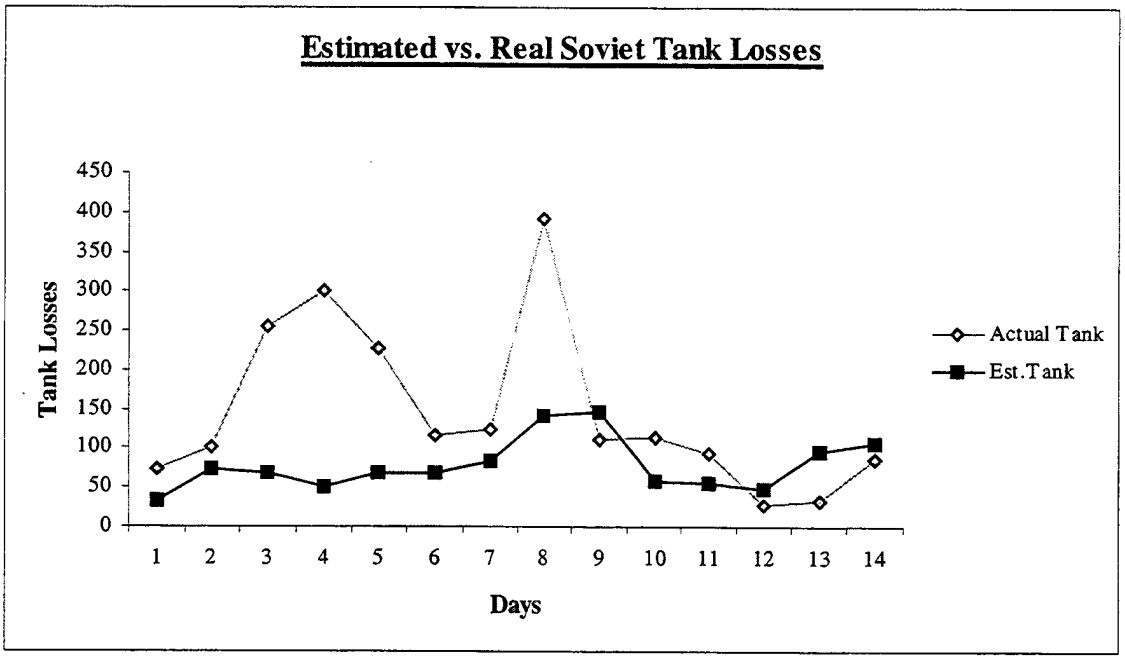

Figure 4.13. Fitted Versus Real Soviet Tank Losses. The model underestimates the battle except for the last three days. Again, day 8 is the heaviest tank battle in history.

Figures 4.8, 4.9, and 4.10 show that this model overestimates the losses for the German forces. The German combat power and the tank power fits are plausible, which are supported by their R^2 values and p-values for the signed-rank test. These fits are better than the ones done in Section 4.2.f using all units data (ACUD). The overestimation for the last five days is because the battle lost its intensity during these days.

Figures 4.11, 4.12 and 4.13 show that mostly the model underestimates the Soviet casualty rates, except for the last two days. This is also supported by the p-values for the Wilcoxon signed-rank test value. The overestimation of the last two days is consistent with the German fits. The highest casualty rate, on day 8, can be explained by historical facts. This particular day is the day that the bloodiest tank battle in history occurred.

In brief, the ATLAS ground attrition model fits to contact combat unit data better than the data when all combat units are considered. The patterns in the fits are similar. R^2 values and p-values for the signed rank test are slightly better in contact units data.

The next section will apply this model to the fighting combat unit data. Later, all of the results of these three data sets are compared.

4. The Application of the ATLAS Ground Attrition Method to Fighting Combat Units Data (FCUD)

This section applies the ATLAS ground attrition process to the fighting combat units data (FCUD) set.

a. Data

The data, FCUD, used in this section was presented in Chapter III.C.6. OH personnel and weapon strengths for both sides and are shown in Tables 3.9 and 3.10 respectively. Personnel and weapon losses of the German and Soviet forces are presented in Tables 3.11 and 3.12 respectively. The data used in this Section includes all units engaged and fighting with the enemy.

b. Combat Power and Force Ratio

The same procedure used in Section 2.b is applied to compute the combat power of the forces. The FCUD is aggregated using the same firepower score values presented in Table 4.2. The computed combat power of the Soviets and the Germans and the attacker's force ratio are presented in Table 4.24. Also, the same combat postures are used in these calculations.

The Germans force ratio for FCUD is greater than the ratios of ACUD and CCUD. Unlike the Germans, the Soviets' FCUD force ratio is smaller than their ACUD and CCUD force ratios. This can be explained by the ratio of the units in contact, and fighting postures.

c. Casualty Rates and its Distribution

The ATLAS ground attrition equations, which are presented in Section 2.d as Equations 4.1 and 4.2, are used to compute the daily estimated casualty rates. The same combat postures as before are used in the equations. Table 4.25 shows the attacker, defender, and the German's and Soviet's daily estimated combat casualty rates. Again,

the attacker always has a higher estimated casualty rate than the defender. The Germans have high estimated casualty rates during the first 4 days.

Day	German Combat Power	Soviet Combat Power	Attacker	Defender's Combat Posture	Attacker Force Ratio
1	567831	221788	German	Prepared	**2.560**
2	568308	379859	German	Prepared	**1.496**
3	469897	445913	German	Prepared	**1.054**
4	491994	369410	German	Prepared	**1.332**
5	457604	426502	German	Hasty	**1.073**
6	461376	394151	German	Hasty	**1.171**
7	507643	521923	German	Hasty	**0.973**
8	554520	616904	Soviet	Prepared	**1.113**
9	580117	450356	Soviet	Prepared	**0.776**
10	438203	514760	German	Hasty	**0.851**
11	465735	216818	German	Hasty	**2.148**
12	258370	213639	German	Hasty	**0.827**
13	320058	96956	Soviet	Prepared	**0.303**
14	396179	306856	Soviet	Prepared	**1.291**

Table 4.24. Estimated Daily the German and the Soviet Combat Power. Notice that the range of force ratio is [0.30-2.56]. On the first day, the force ratio is very close to the 3-1 traditional attacker force ratio.

The Soviets have lower estimated casualty rates than the Germans except on the days they attack. They have almost the same estimated casualty rates on the days they attack.

When the estimated FCUD casualty rates are compared to the estimated ACUD and CCUD casualty rates, it is observed that the FCUD casualty rates are lower than the other two data sets. After having computed casualty rates for both forces, the next issue is to allocate them to different weapon categories.

In order to distribute the combat power casualties to different weapon categories, the algorithm used in Section 4.2.d is also used here. The only difference is, instead of ACUD regression coefficients given in Tables 4.8 and 4.9, the ones presented in Tables 4.26 and Table 4.27 are used. These coefficients are almost the same as the ones

computed using ACUD and CCUD except for the APC and AA coefficients for the Soviets.

Day	Attacker	Attackers Casualty Rate	Defenders Casualty Rate	Germans Casualty Rate	Soviets Casualty Rate
1	German	0.0468	0.0359	**0.0468**	**0.0359**
2	German	0.0469	0.0252	**0.0469**	**0.0252**
3	German	0.0470	0.0206	**0.0470**	**0.0206**
4	German	0.0469	0.0235	**0.0469**	**0.0235**
5	German	0.0390	0.0260	**0.0390**	**0.0260**
6	German	0.0390	0.0274	**0.0390**	**0.0274**
7	German	0.0390	0.0246	**0.0390**	**0.0246**
8	Soviet	0.0470	0.0212	**0.0212**	**0.0470**
9	Soviet	0.0471	0.0176	**0.0176**	**0.0471**
10	German	0.0390	0.0229	**0.0390**	**0.0229**
11	German	0.0388	0.0405	**0.0388**	**0.0405**
12	German	0.0390	0.0279	**0.0390**	**0.0279**
13	Soviet	0.0473	0.0121	**0.0121**	**0.0473**
14	Soviet	0.0471	0.0175	**0.0175**	**0.0471**

Table 4.25. Attacker's Estimated Casualty Rates are Always Higher than the Defenders. The Soviets have lower estimated casualty rates than the Germans except for the days they attack.

In order to test whether or not the German and the Soviet losses are the same, again the regression analysis presented in Section 4.2.e for the FCUD data set is done. The only difference here is that the FCUD data set is used instead of the ACUD data. Table 4.28 shows the results for the regression output for the FCUD data set. The same results occur with the ACUD data set. For the AA losses, it can be concluded that the German and the Soviet losses are not the same, since the p-value is less than 0.05.

Categories	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
Coefficients	**1.00**	**14.29**	**0.87**	**1.34**	**0.20**	**4.44**	**1.01**	**1.00**	**0.61**	**1.18**
Pr(>|t|)	-	0.00	0.00	0.00	**0.32**	0.00	9e-4	7e-4	2e-4	6e-4
R-Squared	-	0.79	0.78	0.78	**0.07**	0.89	0.58	0.60	0.66	0.60
F-statistic	-	9e-6	1e-5	1e-5	**0.32**	1e-7	8e-4	7e-4	2e-4	6e-4

Table 4.26. German Regression Results for Contact Combat Unit Data. The results are not significant only for RKTL weapons.

Categories	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
Coefficients	**1.00**	**6.65**	**1.01**	**1.72**	**0.30**	**2.09**	**1.74**	**1.95**	**2.16**	**0.18**
Pr(>|t|)	-	0.00	1e-4	1e-3	**5e-2**	0.00	0.00	0.00	0.00	0.04
R-Squared	-	0.83	0.69	0.56	0.27	0.83	0.87	0.93	0.88	0.27
F-statistic	-	2e-6	1e-4	1e-3	**5e-2**	2e-6	3e-7	5e-9	1e-7	0.04

Table 4.27. Soviet Regression Results for Contact Combat Unit Data. The results are not significant only for RKTL and AA weapon classes.

	Categories	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA
Personnel	Coefficient	6.65	1.01	1.72	0.30	2.09	1.73	1.95	2.16	0.18
	Pr(>|t|)	0.0	0.0	0.0	5e-3	0.0	0.0	0.0	0.0	0.01
Personnel:IG	Coefficient	7.65	-0.15	-0.37	-0.1	2.35	-0.72	-0.95	-1.55	1
	Pr(>|t|)	0.02	**0.82**	**0.80**	**0.84**	0.02	**0.28**	**0.08**	0.04	**0.008**
R2	-	0.82	0.70	0.60	0.26	0.84	0.86	0.92	0.88	0.4
F-statistic	-	0.0	0.0	0.0	0.01	0.0	0.0	0.0	0.0	2e-4

Table 4.28. The Regression Results for the Hypothesis that the German and Soviet Losses are the Same for the FCUD Data Set.

d. Results

Again, the R^2 values and the figures are used together to compare the actual and estimated casualties. Table 4.29 presents the R^2 values for the Germans and Soviets. The tank and the combat power R^2 values are greater than 0.50, which is a high value for combat models. Also, positive values for the personnel, APC, ATH and the Flame/MG losses of the Germans, and for APC losses of the Soviets occur. Overall, R^2 values for the Germans are better than the ones computed using ACUD and CCUD. For

the Soviets, the APC value is better than the previous ones, but there is no significant difference for the personnel and other weapons. On the following pages, real versus estimated losses are plotted to give a better understanding of the quality of the fits.

As in Sections 4.2.f and 4.3.d, a Wilcoxon signed-rank test is done to test the hypothesis of whether or not the difference between actual and estimated losses is the same. The p-values of the tests are presented in Table 4.30. The difference between the actual and estimated losses for the German tank, artillery, RKTL, ATH, ATL, Flame/MG, and AA weapon groups are accepted as equal to zero at the significant level of 0.05. This result suggests that the model fits better for the FCUD data set than the ACUD and CCUD data sets. This is also supported by the R^2 values. The Soviet p-values for the APC, RKTL, and the AA losses suggest that the model do not under/overestimates the battle for these weapon groups. These results are also supported by the R^2 values.

Weapon groups	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
German	**0.16**	**0.54**	-0.30	**0.35**	-0.13	**0.05**	-0.08	-0.79	**0.00**	-0.25	**0.57**
Soviet	-0.73	-0.21	-0.39	**0.40**	-0.22	-0.81	-1.67	-1.18	-1.67	-0.18	-0.76

Table 4.29. R^2 Values of Personnel Casualties and Weapon Losses. Tank, personnel, APC, ATH, Flame/MG, and combat power values for the Germans and the APC value indicate a better fit. The APC losses fit better for both sides.

Weapon groups	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
German	0.007	**0.06**	**0.15**	0.03	**0.54**	**0.07**	0.01	**0.71**	**0.09**	**0.54**	0.009
Soviet	0.009	0.009	0.02	**0.27**	**0.18**	0.02	0.002	0.002	0.001	**0.35**	0.0012

Table 4.30. The p-Values from the Wilcoxon Signed-Rank Test for the FCUD Data Set. The non-significant values are highlighted.

Figures 4.14, 4.15, and 4.16 show that the ATLAS ground attrition model overestimates the battle on most days for the German forces. The German combat power, personnel and tank fits are plausible which are supported by high R^2 values. In these categories, the model catches the general pattern of the battle for the Germans. These fits are better than the fits of the ACUD and CCUD data sets. This suggestion is also supported by the p-values for the signed rank test.

Figures 4.17, 4.18, and 4.19 show that the model underestimates the Soviet casualties until the ninth day, which is supported by the signed rank test. It fits better in the last four days of the battle. The pattern of the figures is similar to the previous fits presented in earlier sections. The R^2 values and the p-values for the Soviets are consistent.

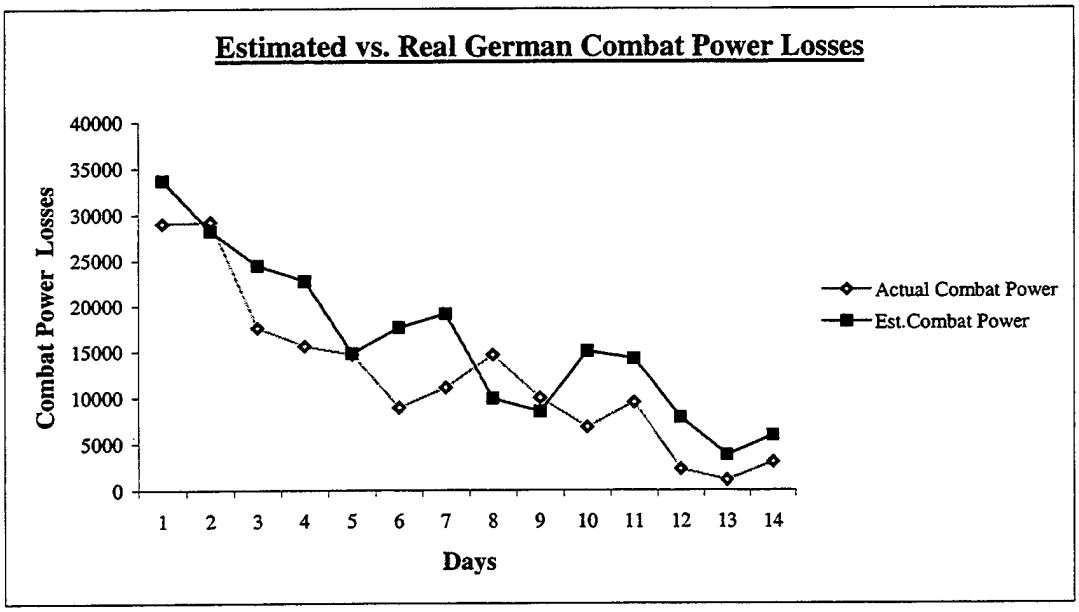

Figure 4.14. Fitted Versus Actual for the German Combat Power Losses Applying ATLAS Attrition Process to Fighting Combat Units Data. On days 2, 5, and 9, the model fits the battle very well.

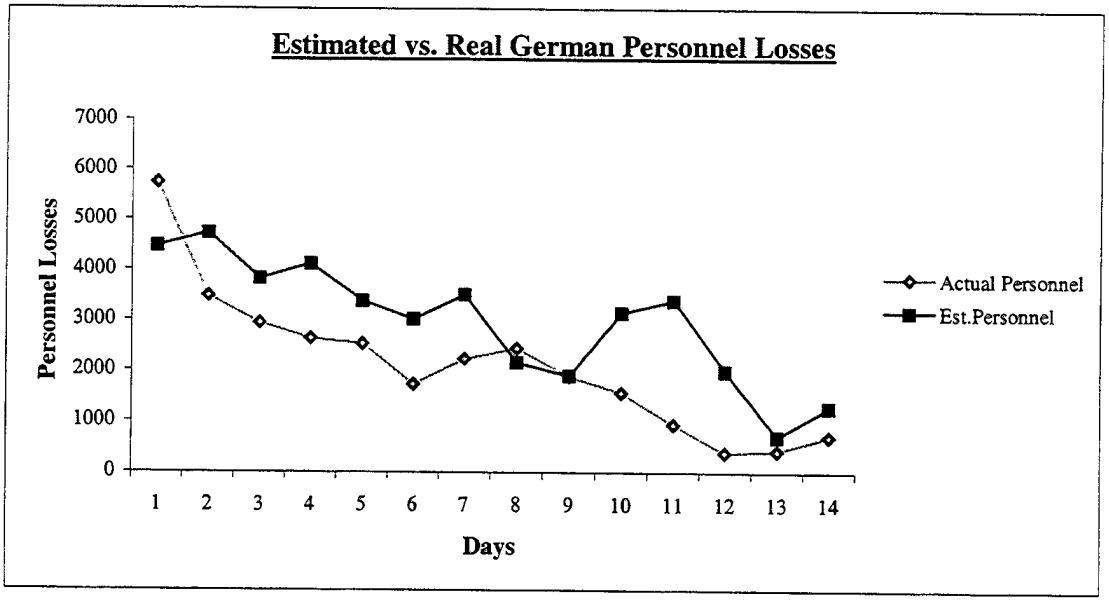

Figure 4.15. Fitted Versus Actual for the German Personnel Losses. The model overestimates battle casualties except for the first, eighth and the ninth days.

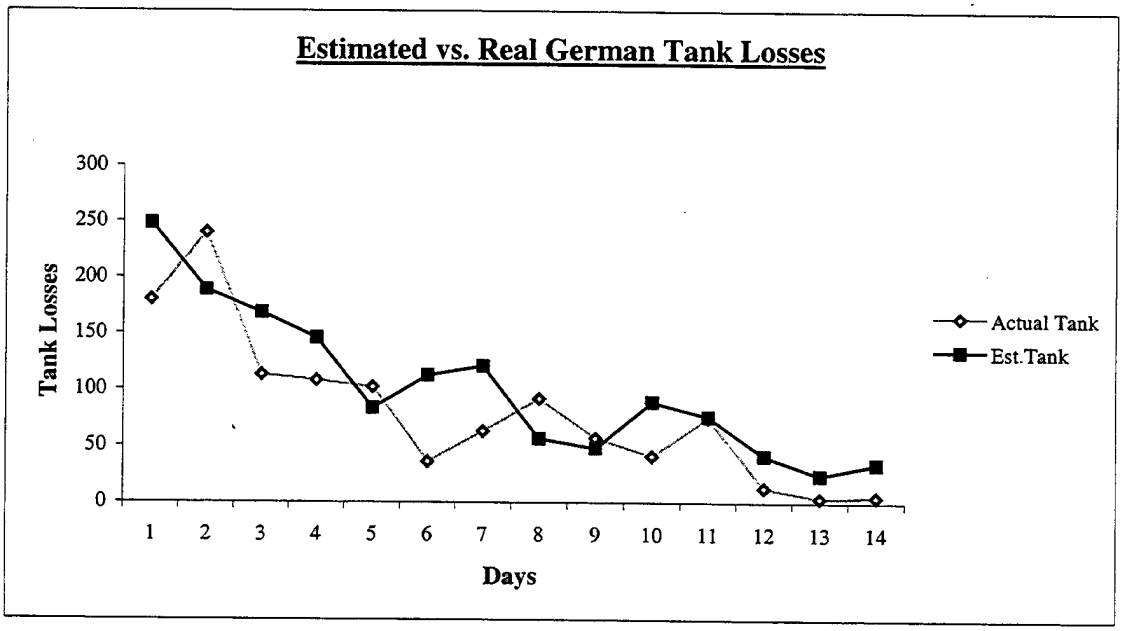

Figure 4.16. Fitted Versus Actual Tank Losses. The trend of the model is fairly good. There is no significant outlier.

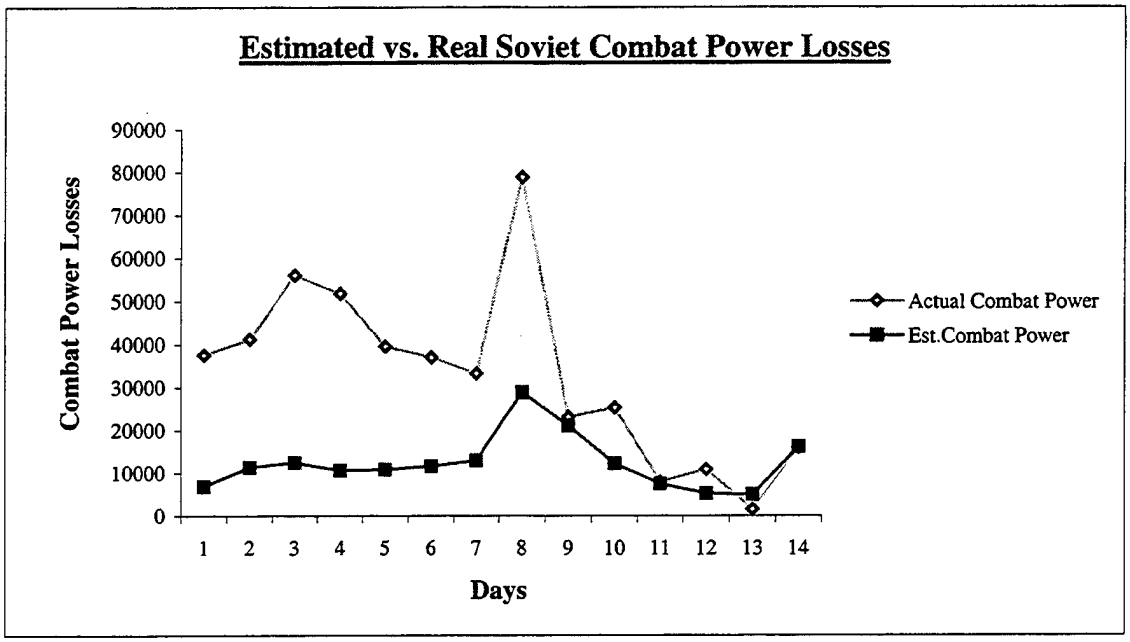

Figure 4.17. Fitted Versus Real for Soviet Combat Power Losses. The model mostly underestimates battle casualties.

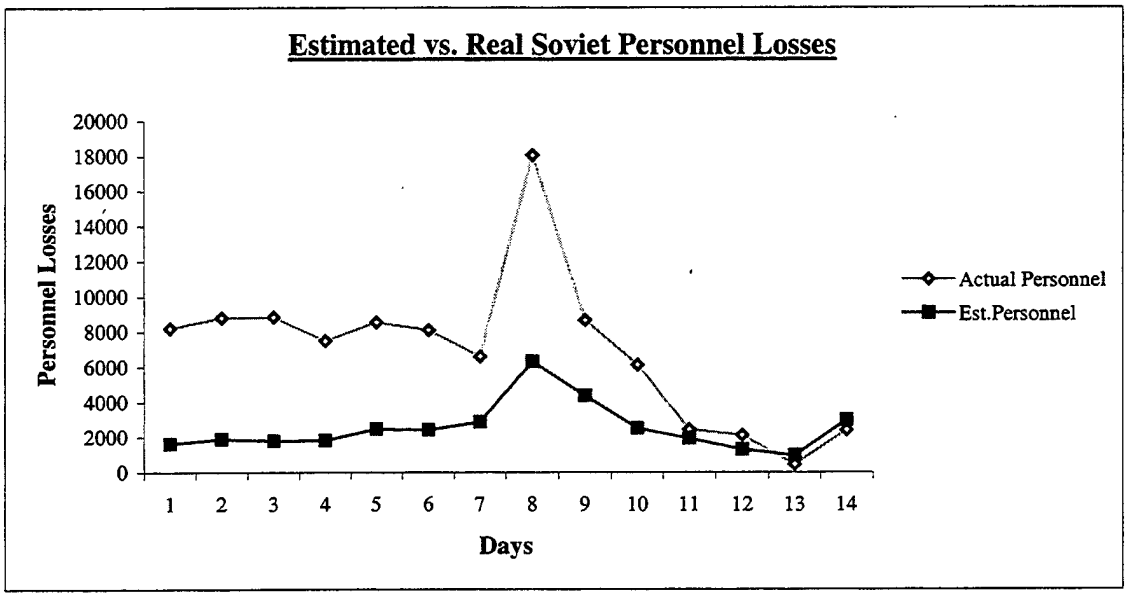

Figure 4.18. Fitted Versus Real Soviet Personnel Losses. The model underestimates battle casualties, but gives a better fit towards the end of the battle.

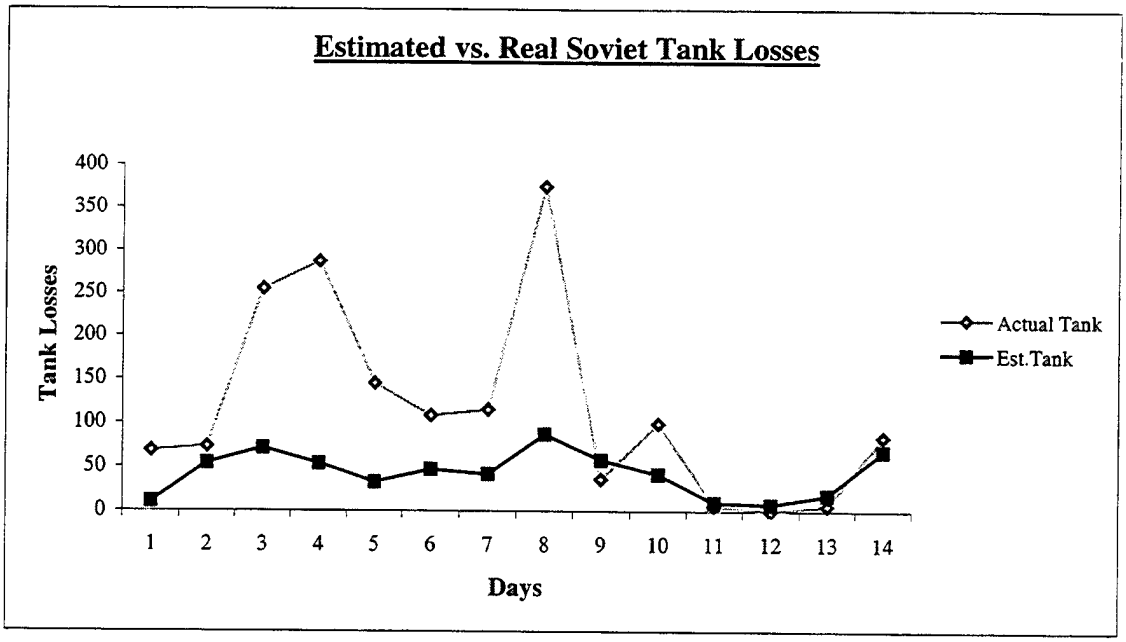

Figure 4.19. Fitted Versus Real Soviet Tank Losses. The model underestimates battle casualties until the ninth day and it fits the battle fairly well on the last four days.

5. The Application of the ATLAS Ground Attrition Method with Air Sortie Data

In this section, air sortie data is added to the model to investigate whether or not it improves the quality of the fits. Basically, the same procedure presented in Sections 2, 3, and 4 is followed except air sorties are added to the Firepower scores.

a. Data

The KDB presents the Soviet and German air sortie activity during the battle of Kursk. The KDB records a daily listing of the number of sorties generated for each aircraft type/mission type combination [Ref. 14:p. 7-1]. The number of onhand aircraft, armament delivered or specific targets attacked is not recorded comprehensively in the KDB. Therefore, the KOSAVE study summarized total daily air sorties according to armed combat roles (attack, bombing, or air-to-air) as well as unarmed roles

(reconnaissance, evacuation). The attack role applies to air attack against specific tactical targets, like contemporary close air support. The bombing role applies to attack against less specific area targets similar to contemporary interdiction [Ref. 14:p. 7-1].

Figure 4.20 shows the total sorties generated in each role for both forces. The Germans generated slightly more total sorties than the Soviets. The Germans generated over twice as many ground attack sorties as the Soviets. There were 1400 German evacuation sorties, but none for the Soviets.

In this study, bombing and ground attack air sorties are included in the models to investigate whether or not adding the air sorties improves the quality of the fits. It is clear that the air sorties affect the attrition in battle. Specifically, the data is for the ground attack air sorties, which provide close fire support to the line units. Table 4.31 presents the air data used in this study.

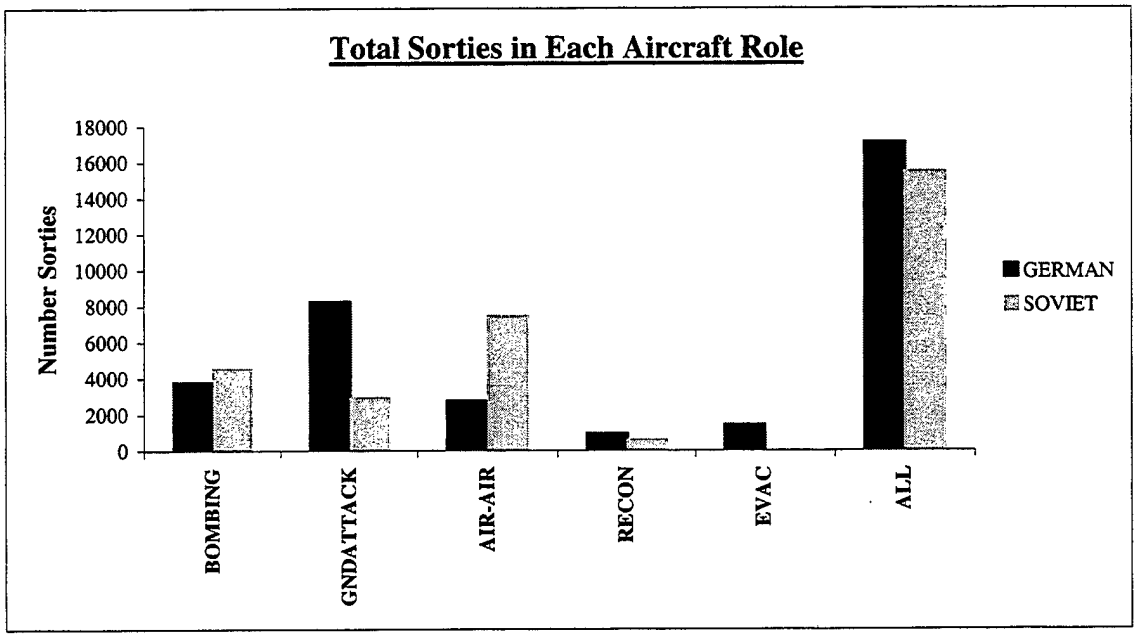

Figure 4.20. Total Sorties in Each Aircraft Role.

Day	GERMAN		SOVIET	
	GND ATTACK	BOMBING	GND ATTACK	BOMBING
1	**1406**	536	354	246
2	**1033**	323	217	396
3	946	553	146	515
4	887	539	388	281
5	882	404	291	181
6	396	134	158	225
7	604	205	152	196
8	398	62	272	331
9	342	109	257	366
10	645	502	124	580
11	259	282	122	247
12	248	30	181	500
13	69	53	159	177
14	0	18	94	190

Table 4.31. The Daily Number of Ground Attack and Bombing Air Sorties for Both Sides. The Germans generated more ground attacks during the first two days.

The three data sets presented in Chapter III and used in Sections 2, 3, and 4 are also used here. Thus, personnel and weapon numbers do not change. Only the air sortie data shown in Table 4.31 is added to these three data sets. No classification is made for air sortie data, i.e., contact or fighting air sorties.

b. Combat Power

As presented in Table 4.1, attack helicopters have an average firepower score value of 7. Although helicopters were not used in World War II, they are the only air vehicles that firepower score values are assigned to in RAND's proposed ground force scoring system. Firepower score values for the ground attack and bombing air sorties relative to RAND's scores for the attack helicopters were assigned. Thus, ground attack air sorties have the score value of 150, and the bombing air sorties have the firepower

score value of 120. Firepower score values of all weapons used in this section are presented in Table 4.32. Combat powers and force ratios are computed with the same methods described in Sections 2.c and 2.d.

Weapon Groups	Prsnl.	Tank	APC	ARTY	RKTL	ATH	MTR	ATL	FLAME /MG	AA	Grnd. Attack	Bomb ing
Firepower Scores	1	100	20	80	90	25	20	20	10	25	150	120

Table 4.32. Firepower Score Values for 11 Weapon Groups. These scores are computed relative to RAND's firepower score values.

c. Casualty Rates and its Distribution

Again, ATLAS ground attrition equations (Equations 4.1 and 4.2) are used to compute the daily casualty rates. The same procedure is followed to distribute the combat power casualty rates to weapon categories. The only difference is that air sortie data is not used in the casualty distribution. Since KV scoreboards are not available in the database, we do not know the ground casualties due to the air sorties. The air sortie data is included only in the computation of the aggregated combat power of the forces.

d. Results

Figures and R^2 values are used to compare the actual and estimated casualties. Table 4.33 presents the R^2 values for the Soviets and Germans. In the ACUD data set, positive values for the Soviet tank and APC values and for the German tank and combat power values occur. In this data set, adding the air sorties to the model improves the quality of the fits. In the CCUD data set, Soviet APC, German tank, APC, and combat power values are positive. Again, adding the air sorties gives better R^2 values also for this data set. In the FCUD data set, the German personnel, tank, APC, ATH, Flame/MG

and combat power, and Soviet APC R^2 values are positive. The R^2 values are better than the ones computed without air sortie data

According to the R^2 values for both sides in all data sets, adding the air sorties to the ATLAS ground attrition model improves the quality of the fits. However, the figures should be considered before making this conclusion.

Data Set	Forces	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
ACUD	GERMAN	-0.62	**0.24**	-0.42	-0.05	-0.08	-0.44	-0.39	-0.25	-0.30	-0.11	**0.11**
	SOVIET	-0.24	**0.03**	-0.47	**0.14**	-0.42	-0.73	-0.82	-0.63	-0.75	-0.19	-0.23
CCUD	GERMAN	-0.67	**0.33**	-0.46	**0.01**	-0.09	-0.39	-0.54	-0.25	-0.27	-0.09	**0.21**
	SOVIET	-0.26	-0.15	-0.40	**0.10**	-0.13	-0.68	-1.01	-0.81	-1.07	-0.24	-0.44
FCUD	GERMAN	**0.23**	**0.58**	-0.15	**0.40**	-0.10	**0.17**	-0.06	-0.28	**0.06**	-0.06	**0.62**
	SOVIET	-0.40	-0.08	-0.26	**0.46**	-0.15	-0.51	-0.98	-0.70	-1.00	-0.14	-0.42

Table 4.33. R^2 Values for both Sides in Different Data Sets with Air Sorties. The APC values are almost positive in all models for both sides. The only positive personnel value is seen for the Germans in the FCUD data set. The German combat power values are positive in all three data sets.

Data Set	Weapon groups	Prsnl	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
ACUD	German	9E-4	5E-3	0.02	0.02	**0.05**	5E-3	0.02	**0.34**	0.02	**0.14**	1E-3
	Soviet	**0.21**	**0.46**	**0.32**	**0.90**	**0.05**	**0.12**	**0.09**	**0.1**	**0.05**	**0.42**	**0.12**
CCUD	German	9E-4	0.01	**0.07**	0.02	**0.6**	7E-3	0.02	**0.5**	0.03	**0.24**	7E-3
	Soviet	0.03	0.04	**0.30**	**0.62**	**0.05**	**0.21**	0.04	0.04	**0.05**	**0.9**	0.03
FCUD	German	5E-3	**0.06**	**0.15**	0.03	**0.5**	**0.07**	0.01	**0.67**	**0.09**	**0.54**	8E-3
	Soviet	6E-4	8E-3	0.02	**0.32**	**0.2**	0.02	2E-3	2E-3	1E-3	**0.35**	1E-3

Table 4.34. The p-Values from the Wilcoxon Signed-Rank Test for all Data Sets for Both Sides. The non-significant values are highlighted.

Figures 4.21 and 4.22 show the estimated versus actual combat power losses in the FCUD data set for the Germans and Soviets. When comparing these figures with the ones presented in Figures 4.8 and 4.11, which show the fits without air sortie data, it is seen that patterns of the curves are exactly the same. Although, the air sorties improve the quality of the R^2 values, they do not improve the quality of the curves significantly. This is true for personnel and all weapon groups in all data sets.

As in previous models, a non-parametric Wilcoxon signed-rank test is done to test the hypothesis of whether or not the difference between actual and estimated losses is the same. The p-values of the tests are presented in Table 4.34. The difference between the actual and estimated losses for the German RKTL, ATL, and AA weapon groups are accepted as equal to zero at the significant level of 0.05 for all three of the data sets. In the FCUD data set, the German tank, artillery, RKTL, ATH, ATL, Flame/MG, and AA weapon groups have a greater p-value than the significant level. This result suggests that the model fits better for the Germans for the FCUD data set than the ACUD and CCUD data sets, which is also supported by the R^2 values.

The difference between actual and estimated losses for all Soviet weapon groups is equal zero at the significant level of 0.05 in the ACUD data set. In the CCUD data set, the Soviet artillery, APC, RKTL, ATH, Flame/MG, and AA weapon groups have a greater p-value than the significant level. The Soviet p-values for the APC, RKTL, and the AA losses are not significant in the FCUD data set.

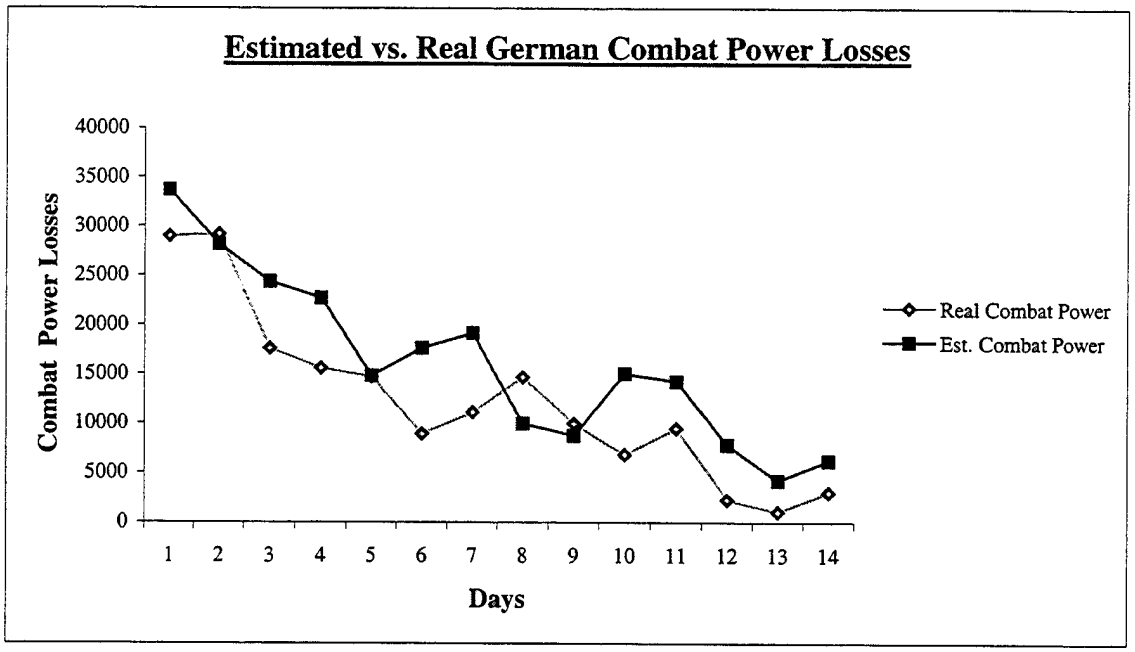

Figure 4.21. Fitted Versus Actual for the German Combat Power Losses with Air Sorties in the FCUD Data Set. The figure has the same pattern as Figure 4.2. The model's trend is good, but overestimates in some parts, especially towards the end.

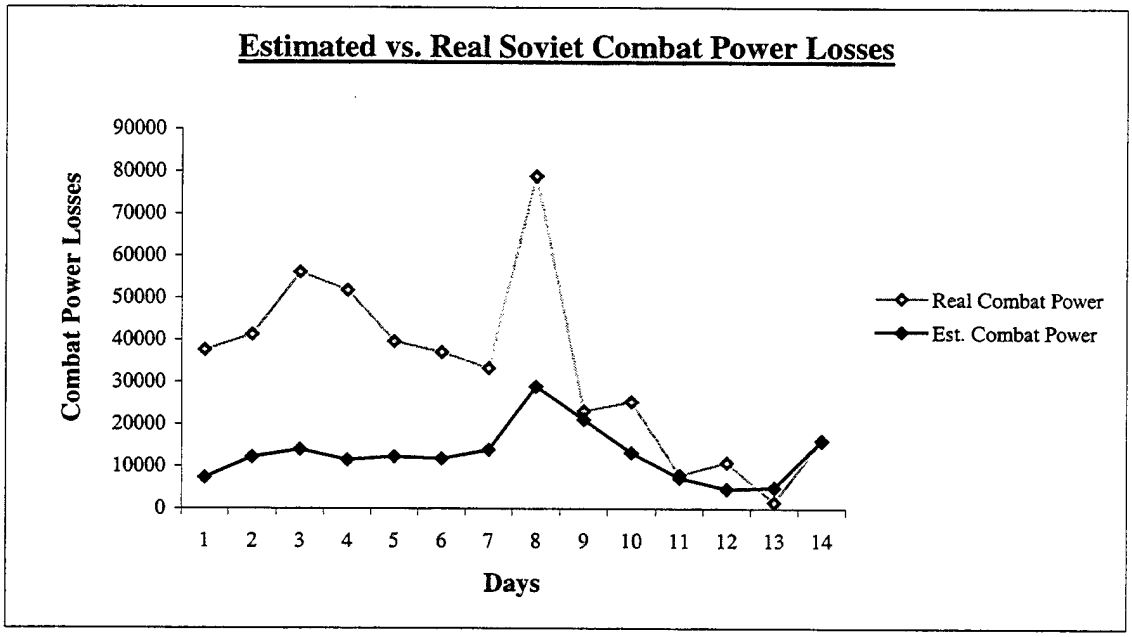

Figure 4.22. Fitted Versus Actual Soviet Combat Power Losses with Air Sorties in the FCUD Data Set. The figure has the same pattern as Figure 4.5. The model underestimates battle casualties.

6. The Application of the ATLAS Ground Attrition Method Using Different Firepower Score Values

In this section, different firepower score values are used in the models. Bracken [Ref. 8] used firepower score values (in his study they are called weight) in aggregation of the total forces in his study. He used personnel strength, APCs, tanks, and artillery in his models and they are weighted by 1, 5, 40, and 20 respectively. Bracken [Ref. 8] states in his study that, "The weights given above are consistent with those of studies and models of the U.S. Army Concepts Analysis Agency. Virtually all theater-level dynamic combat simulation models incorporate similar weights, either as inputs or as decision parameters computed as the simulation progresses."

According to Bracken's weights, an artillery piece is twice as valuable as a tank. The tank, APC, and artillery score values were assigned the same as Bracken's. For other weapon's firepower score values, RAND's values were used. See Table 4.1 as a reference. Since Bracken did not use other weapon groups in his study, the weapons' weights are the same as the ones used in previous sections. In Table 4.1, APC, MTR, ATL have the same score (i.e., 20). In Table 4.35, APC has a score of 5 from Bracken's weights, so MTR and ATL have the same score value of 5. The ATH, AA, and Flame/MG scores are assigned according to the proportion of APC value in Table 4.1. The RKTL score is given relative to the artillery value. In Table 4.32, the ground attack score is 50% greater than the highest weapon score, which is tank in our scores and was used in previous sections. In Bracken's weights, the artillery has the highest score value. Therefore, the ground attack score value was assigned relative to the artillery score.

Bombing score values are defined relative to the ground attack value as in Table 4.32. Table 4.35 shows the firepower score values used in this section.

Weapon Groups	Prsnl.	Tank	APC	ARTY	RKTL	ATH	MTR	ATL	FLAME /MG	AA	Grnd. Attack	Bomb ing
Firepower Scores	1	20	5	40	50	8	5	5	2.5	8	60	50

Table 4.35. Firepower Score Values for 11 Weapon Groups. Tank, APC, and Artillery Values are from Bracken's Study. Others are computed relative to RAND's values.

The data used in this section is the same as that of the previous section, and air sorties are also included. These weights are applied to the three data sets: ACUD, CCUD, and FCUD. The same procedures are used to compute the combat power, force ratios, casualty rates and to distribute the casualty rates into different weapon groups. Table 4.36 shows the R^2 values for both forces in three data sets. When these results are compared with the results given in Table 4.33, Bracken's weights do not give a better fit except for the Soviet values in the CCUD data set, which are slightly better.

Data Set	Forces	Personnel	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
ACUD	**GERMAN**	-0.76	**0.18**	-0.66	-0.15	-0.13	-0.69	-0.42	-0.81	-0.39	-0.26	-0.28
	SOVIET	-0.50	-0.08	-0.62	**0.09**	-0.82	-1.06	-1.39	-1.03	-1.32	-0.20	-0.57
CCUD	**GERMAN**	-0.81	**0.28**	-0.71	-0.04	-0.14	-0.64	-0.55	-0.86	-0.38	-0.32	-0.24
	SOVIET	-0.21	-0.12	-0.38	**0.10**	-0.12	-0.64	-0.92	-0.74	-0.99	-0.23	-0.34
FCUD	**GERMAN**	**0.16**	**0.53**	-0.32	**0.36**	-0.14	**0.03**	-0.07	-0.85	**0.00**	-0.27	**0.45**
	SOVIET	-0.63	-0.14	-0.34	**0.44**	-0.19	-0.70	-1.50	-1.04	-1.50	-0.16	-0.71

Table 4.36. R^2 Values for Both Sides in Different Data Sets with Air Sorties Using Bracken's Weights.

Figures 4.23 and 4.24 show the estimated versus actual combat power losses in the CCUD data set for the Germans and Soviets. Comparing these figures with Figures

4.21 and 4.22, it is clear that using different weights do not change the pattern of the fits significantly. The only remarkable difference is on the ninth day. On this day, for the Germans, the model overestimates the battle, while there is a better fit for the Soviets. Again, the p-values of the Wilcoxon signed-rank test are presented for the null hypothesis that the difference between the actual and estimated losses is equal to zero. The results are presented in Table 4.37.

Data Set	Weapon groups	Prsnl.	Tank	ARTY	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
ACUD	German	6E-4	5E-3	0.04	0.02	**0.6**	5E-3	0.02	**0.4**	0.02	**0.15**	6e-4
	Soviet	**0.17**	**0.54**	**0.26**	**0.95**	0.03	**0.1**	**0.08**	**0.09**	0.03	**0.4**	0.2
CCUD	German	7E-3	**0.05**	0.04	**0.05**	**0.15**	0.04	0.03	**0.18**	**0.06**	**0.21**	9e-4
	Soviet	0.03	**0.06**	**0.3**	**0.6**	**0.08**	**0.24**	**0.05**	**0.07**	**0.1**	**0.9**	0.04
FCUD	German	9E-3	**0.07**	**0.21**	0.03	**0.5**	**0.09**	0.01	**0.7**	**0.09**	**0.5**	5E-3
	Soviet	1E-3	0.01	0.02	**0.32**	**0.22**	0.03	2E-3	3E-3	1E-3	**0.4**	1E-3

Table 4.37. The p-Values from the Wilcoxon Signed-Rank Test for all Data Sets for Both Sides. The significant values are highlighted.

The difference between actual and estimated losses for the German RKTL and AA weapon groups equals zero at the significant level of 0.05 for all of the three data sets. In the ACUD data set, the German tank, APC, RKTL, ATL, Flame/MG, and AA weapon groups have a greater p-value than the significant level. For the FCUD data set, most of the German weapon groups have greater p-values than the significant level.

The difference between actual and estimated losses for most of the Soviet weapon groups equals zero at the significant level of 0.05 in the ACUD and CCUD data

sets. In the FCUD data set, only the Soviet artillery, APC, RKTL, and AA weapon groups have a p-value greater than the significant level.

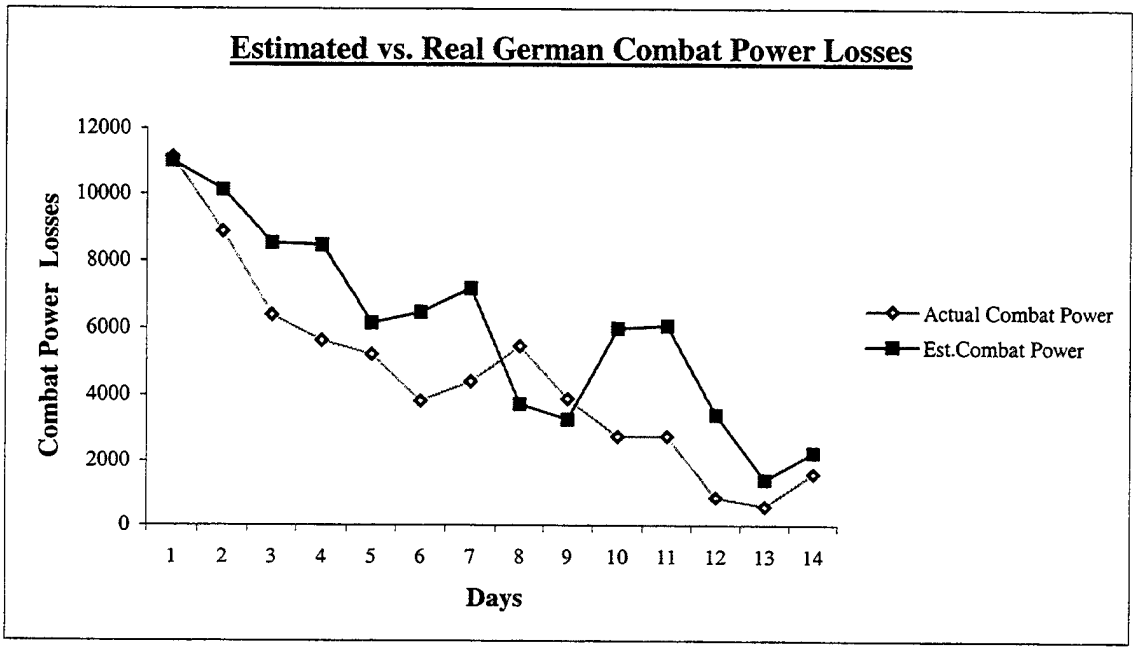

Figure 4.23. Estimated Versus Actual German Combat Power Losses with Air Sorties Using Bracken's Weights for the FCUD Data Set. The model overestimates battle casualties specifically after the 8th day of the battle.

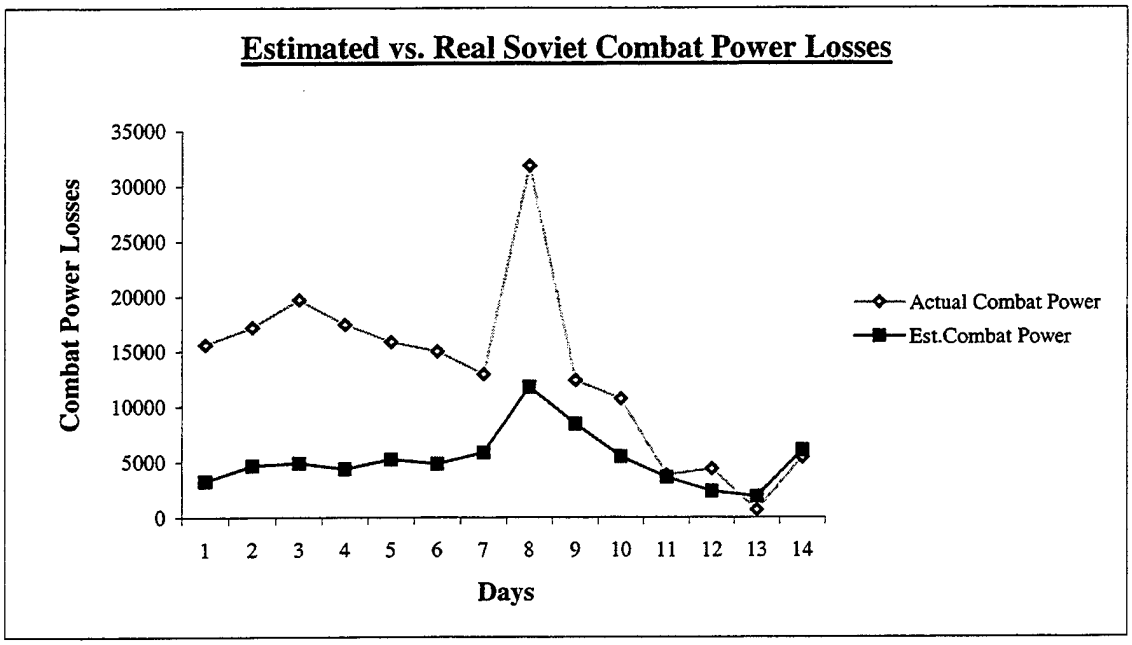

Figure 4.24 Estimated Versus Actual Soviet Combat Power Losses with Air Sorties Using Bracken's Weights for the FCUD Data Set. The model underestimates most of the battle except for the last four days.

B. APPLICATION OF RAND'S SITUATIONAL FORCE SCORING METHODOLOGY

In this section, RAND's situational force scoring (SFS) methodology [Ref. 7] is applied to the data on the Battle of Kursk. Patrick Allen presented the SFS methodology in 1991 as an initial attempt to describe a framework for situational adjustments to ground force scores.

In combat in the real world, different types of weapons perform better or worse in different types of terrain and engagements. Furthermore, each weapon type is more or less effective depending upon the mix of weapons of both sides' forces. The SFS methodology defines a series of multipliers that can be applied to any existing force scoring mechanisms. One of the objectives of this method is to improve the

representation of ground force close combat in aggregate combat models that use scores of one form or another to compute force ratio, attrition, and movement as a result of combat. SFS seeks to accomplish this objective by adjusting the scores, i.e., weapon system values, dynamically to reflect the effects of the type of terrain, type of battle, and combined arms imbalances or shortages on each side's effective force scores [Ref. 7].

The SFS consists of a 20-step calculation process divided into four stages that adjust force scores for a variety of factors. All of the steps require simple calculations, such as addition, subtraction, multiplication and division. The four stages are defined in the study as:

- Varying the strength of each category of weapon as a function of the terrain and type of engagement
- Modifying category multipliers to account for shortages in the combined arms mix
- Calculating combat outcomes, including losses to each side and the FLOT (forward line of own troops) movement rate
- Calculating the casualty distribution across each category of weapon.

In this study, the SFS methodology is applied to our data sets by following the 20 steps presented in Tables 2.1, 2.2, and 2.3 of RAND's notes [Ref. 7:pp.9-10]. Although, the method needs simple calculations, the hard part is to determine the values of the multipliers used in the computations. All the details of the calculations will not be discussed, but the necessary information will be provided for future analyses.

First, all combat units data (ACUD) is used in the computations and the steps will be explained in this data set. Later, the results are presented for both contact combat unit

data (CCUD) and fighting combat unit data (FCUD). The application of SFS method will be presented in four stages as mentioned in the RAND's note [Ref. 7].

1. Varying Asset Strength

This section presents the first seven steps of the algorithm described in Table 2.1[Ref. 7:p. 9]. The SFS method starts with the number of assets in each weapon category in the force, and three multipliers are applied to obtain the situational strength of each weapon category as a function of the combat situation.

The first multiplier is used in Step 2 [Ref. 7:p.15] for scoring assets. The SFS methodology assumes the use of an existing scoring methodology that already defines a relative value between combat assets. As mentioned in Section A.1, RAND's proposed scoring methodology was used [Ref. 7:Appendix E] as a baseline for our firepower score values. Again, the same firepower score values given in Table 4.2 are used herein. The 10 weapon categories used in Section A are aggregated by these firepower score values into 6 categories. Air defense weapons (AA) are not used. The categories are as follows:

- Tanks: tanks
- APC: APC
- Long-range anti-armor (LR atgms.) weapons: Heavy antitank weapons (ATH)
- Short-range anti-armor (SR atgms.) weapons: Light antitank weapons (ATL)
- Infantry: Personnel, mortar, Flame/MG
- Artillery: Artillery, rocket launchers (RKTL)

The second multiplier, force score multiplier, is defined in Step 4 [Ref. 7:p.16]. This multiplier is used to account for some of the more qualitative factors that influence

combat effectiveness. Some of these factors include the effects of the unit's level of training, cohesiveness, or nationality [Ref. 7:p.16]. On the Eastern Fronts of the two World Wars, the German relative combat effectiveness superiority over the Russians generally ranged between the factors of 2.0 and 3.0. [Ref. 3: p.43]. The upper limit 3 was used for this multiplier for the Germans, because in Turkes's fits [Ref. 11] of the square law, he estimated the Germans were 3.4 times as effective per man as the Soviets.

The third multiplier is the situational category multiplier, which is defined in Step 6 [Ref. 7:p.17]. Two look-up tables were developed in the SFS methodology shown in Tables 3.1 and 3.2 [Ref. 7:p.18-19]. One is for the attacker and one for the defender. Given the type of battle and the type of terrain, weapon category multipliers are determined from these look-up tables. For the type of battle, the same combat postures defined in Section B were used. The data used in this study includes the units in the southern front of the battle. Since, all units in this sector were aggregated, it is not easy to determine the type of terrain in the battle. For this reason, the mix type of terrain was used in this method.

The result of this stage is the situational category strength contributed by each category weapon as a function of terrain and type of battle, before any combined arms shortages are calculated.

2. Determining Shortage Category Multipliers

In this stage, shortage category multipliers are defined. The weapon categories used in the first stage are mapped into the three combined arms branches: armor, infantry, and artillery using Tables 4.1 and 4.2 [Ref. 7:pp.31-32]. After mapping the weapon

categories, the next step is to determine whether or not there are any shortages as a function of the combat situation. The procedure for determining whether or not a shortage exists in a platform is described in RAND's note [Ref. 7:p.33].

Due to the unavailability of the necessary information in our data, it was not possible to determine the shortage category multipliers. Therefore, in this step the value of 1 was used for the shortage category multipliers, which indicates that they do not have any effect on the calculations. Moreover, there is no evidence from the historical accounts or the data that suggest a shortage or imbalance in the weapon groups for both the Germans and the Soviets.

The result of this stage on the final category strength is the same with the situational category strength computed at the end of the previous stage.

3. Combat Assessment

In this stage the force ratio is computed. The force ratio, or the "situationally adjusted" or "modified" force ratio (MFR), is the ratio of attacking force strength to defending force strength. The force ratio and the type of battle together determine the loss rates for each side and the FLOT movement rate [Ref. 7:p.39].

The first step in determining the loss and movement rates is to determine the intensity of the attack. Three levels of intensity are defined in this methodology, which are high, medium, and low. The intensity levels assigned to each day for the Battle of Kursk are presented in Table 4.38.

Once the intensity levels of the battle are defined, the casualty and FLOT movement rates are computed by using the SFS equations [Ref. 7:p. 41]. Table 4.38 shows the force ratio, combat posture, battle intensity, attacker's preparation days and the estimated combat power casualty rates for each side for the ACUD data set.

The German estimated combat casualty rates are very high during the first four days due to their low force ratios on those days. The Soviet estimated combat casualty rates on the eight and ninth days are very high due to their very low force ratios as an attacker in those days. Also, on the last two days, the Soviet rates are high.

When the force ratio given in Table 4.38 is compared with the ones computed in the ATLAS model (Table 4.5), the SFS force ratios are greater than the ones for ATLAS. The interesting result is that the Soviet force ratios are four times lower than the ATLAS ones on the days they attacked. This can be explained by the Relative Combat Effectiveness of the Germans over the Soviets with a value of 3. In the ATLAS algorithm, this factor is not implemented explicitly, so the Soviet force ratio on the days they attack was greater than the Germans. The jump in the force ratio on the 4^{th} day is because of the decrease of the Soviet assets. After the 4^{th} day, the German force ratio is greater than 2. The first reason for this is the decrease in Soviet personnel onhand and weapon numbers. The second and primary reason, is the change in combat posture type from prepared to hasty. In real combat, it is not possible to predict the battle's intensity, which is an important multiplier in the attrition, but in our model the battle intensity is tuned to reflect the real battle.

Day	Attacker	Defender's Combat Posture	Level of Battle Intensity	Attacker's Preparations	Attackers Force Ratio	Estimated German casualty rates	Estimated Soviet casualty rates
1	German	Prepared	Medium	more than 7	0.8786	**0.1374**	**0.0262**
2	German	Prepared	Medium	more than 7	0.8563	**0.1408**	**0.0258**
3	German	Prepared	Medium	more than 7	0.8753	**0.1379**	**0.0262**
4	German	Prepared	Medium	more than 7	0.9362	**0.1295**	**0.0273**
5	German	Hasty	Medium	more than 7	2.0979	**0.0612**	**0.0458**
6	German	Hasty	Medium	1 to 2	2.1861	**0.0511**	**0.0353**
7	German	Hasty	Medium	1 to 2	2.2441	**0.0499**	**0.0359**
8	Soviet	Prepared	High	less than 1	0.4404	**0.0164**	**0.3083**
9	Soviet	Prepared	Medium	1 to 2	0.4308	**0.0125**	**0.2316**
10	German	Hasty	Low	1 to 2	2.2180	**0.0182**	**0.0107**
11	German	Hasty	Low	1 to 2	2.2042	**0.0183**	**0.0106**
12	German	Hasty	Low	1 to 2	2.2368	**0.0180**	**0.0107**
13	Soviet	Prepared	Low	1 to 2	0.4137	**0.0037**	**0.0866**
14	Soviet	Prepared	Low	1 to 2	0.4082	**0.0036**	**0.0876**

Table 4.38. Force Ratio, Estimated Combat Casualty Rates for Each Side in ACUD Data Set. Notice that for the first four days, the German estimated casualty rates are very high. The Soviets have higher estimated casualty rates on the days they attacked.

The German estimated combat casualty rates are lower than the ATLAS ones except for the first four days. Unlike the Germans, the Soviet estimated combat casualty rates are higher than the ATLAS ones except for days 11 and 12.

4. Casualty Distribution

After the combat results have been assessed, the losses to each side are distributed among the assets of each force. As mentioned in Section A, one of the problems with traditional force-on-force models is that the casualty rates for each category of weapons are the same as the overall combat casualty rates. From historical studies and a wide range of higher-resolution games and simulations, it is clear that the loss rates tend to be higher for armor assets and lower for artillery assets. In order to place this type of information in a form useful for purposes of extrapolation, RAND estimated a category loss multiplier for each category of weapons. [Ref. 7:p.44]

These category loss multipliers are presented in Table 6.1 in RAND's notes [Ref. 7:p.46]. In this table, multipliers are defined according to a defender's combat posture and battle's primary assault weapon (armor or infantry). The same combat posture for a defender given in Table 4.38 was used. Since the Battle of Kursk was a major tank battle, armor was used for the battle's primary assault weapon. Once the category loss multipliers are determined, the combat power casualty rate given in Table 4.38 is distributed to the armor, infantry, and artillery weapon categories. The results are given in the following section.

5. Results

Although the SFS starts with different types of weapon categories, in the final stage, the combat power casualty is distributed to only armor, artillery, and infantry weapon groups. No category loss multipliers are defined for more specific weapon groups, such as APC, mortar or antitank. As a result, the actual versus real armor, infantry, and artillery losses were compared. These categories consist of the following weapon groups:

- Armor: Tank, APC
- Infantry: Personnel, mortar, ATH, ATL, Flame/MG
- Artillery: Artillery, RKTL

Again R^2 values and comparison figures are used to show the results of this algorithm. The R^2 values are shown in Table 4.39 for all of the data sets together. The non-parametric Wilcoxon signed-rank test to test the hypothesis of whether or not the difference between the actual and estimated losses is the same is also used in this method. The p-values of the tests are presented in Table 4.40.

The German armor R^2 value in ACUD, and combat power value in FCUD data indicates a better fit. When the three data sets are compared, the SFS method fits the FCUD data better than the other two data sets for both forces. Despite having broad "tuning parameters," such as relative combat effectiveness factor, intensity, and the number of day for attacker's preparation, the SFS does not give a good fit. The R^2 values for the infantry and artillery groups specifically are very poor. The comparison of combat power values to the ATLAS combat power values shows that the SFS did not fit the data on the Battle of Kursk better than the ATLAS algorithm.

Data Sets	Weapon Groups	Armor	Infantry	Artillery	Combat Power
ACUD	German	**0.59**	-180.40	-131.54	-13.45
	Soviet	-0.31	-48.15	-69.36	-16.82
CCUD	German	**0.066**	-32.47	-25.51	-0.51
	Soviet	-0.52	-48.85	-35.38	-17.76
FCUD	German	-0.28	-7.79	-9.40	**0.38**
	Soviet	-0.04	-43.73	-28.09	-14.45

Table 4.39 R^2 Values for both Sides in the Three Data Sets. The value of German armor in the ACUD data and the combat power value in FCUD data are very plausible. Other values are very poor, mostly negative and very low.

Data Sets	Weapon Groups	Armor	Infantry	ARTY	Combat Power
ACUD	German	**0.24**	2E-4	0.001	0.01
	Soviet	**0.21**	**0.3**	0.03	**0.95**
CCUD	German	2E-4	6E-4	4E-3	**0.11**
	Soviet	0.21	0.62	0.03	**0.95**
FCUD	German	1E-4	0.01	0.03	**0.6**
	Soviet	**0.17**	**0.95**	**0.10**	**0.90**

Table 4.40. The p-Values from the Wilcoxon Signed-Rank Test for all Data Sets for both Sides. The non-significant values are highlighted.

Figure 4.25 shows that the model highly overestimates the first four days of the battle for German combat power losses in the ACUD data. It gives a better fit after the eight day of the battle. Overall, the p-value for its signed rank test shows that the difference between actual losses is not zero. In the CCUD and FCUD data sets, the big difference in the first four days gets smaller towards the end of the battle. Their signed rank test p-values suggest that the difference between actual and estimated values is zero. As a result, the model gives better combat power R^2 values in these data sets. Figures 4.26, 4.30 and 4.34 show the German armor losses comparison for the ACUD, CCUD, and FCUD data sets respectively. The model fits better in the ACUD data set for the armor losses, which is supported by its R^2 values and signed rank test p-values. The model under/overestimates the battle for CCUD and FCUD data sets which is seen from their signed rank test values. Overall, the model gives a good pattern for the armor losses in all three data sets.

Figures 4.27, 4.31 and 4.35 shows that the Soviet combat power fits the data good except for days 8, 9, 13 and 14 in which the Soviets have very low force ratios. Due to the large differences on these days, the Soviet R^2 values are negative and very small. The Wilcoxon signed-rank test p-values for the Soviet combat power losses are very high which strongly suggest that the difference between actual and estimated losses is zero at the significant level 0.05. These facts prove that the R^2 value by itself is not enough to show the quality of the fits. Thus, the R^2 value, figures and signed rank test should be considered together in comparison with the different models.

The Soviet combat power losses show a consistent pattern in all data sets. Likewise, the Soviet armor losses also have a consistent pattern in all data sets. They have a better fit than the German combat power losses. In all data sets, the Soviet armor losses have high p-values, which suggest that the real and estimated loss difference is zero. The model fits poorly for infantry and artillery losses in all three data sets for both sides.

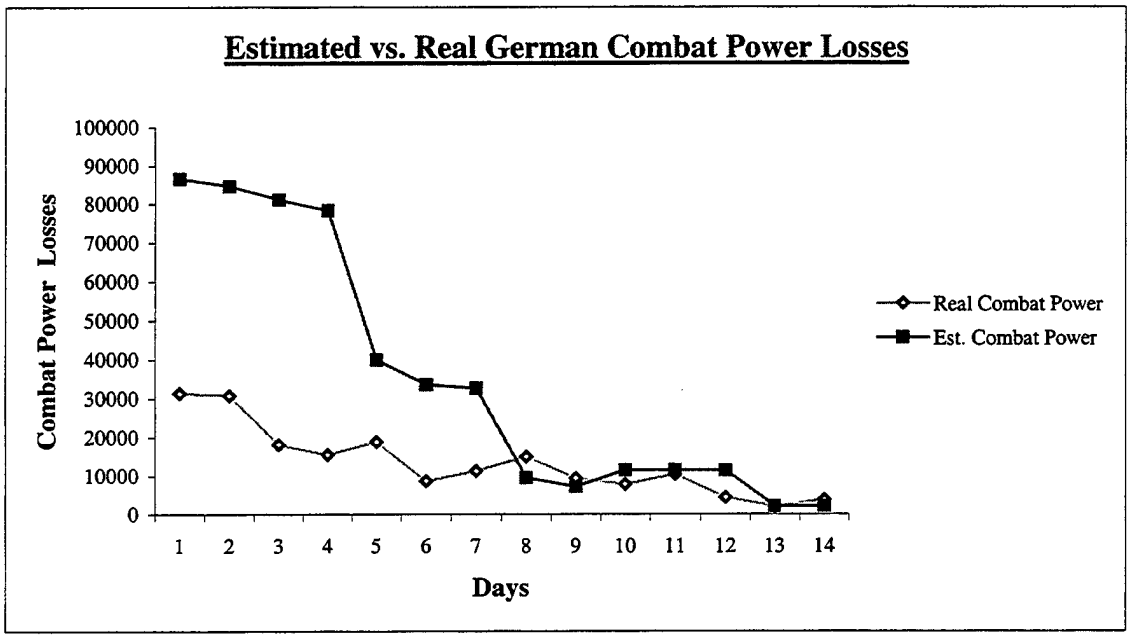

Figure 4.25. Estimated Versus Actual German Combat Power Losses in the ACUD Data. The model dramatically overestimates the casualties until the eight day. The high overestimation during the first four days is due to the lower German force ratios.

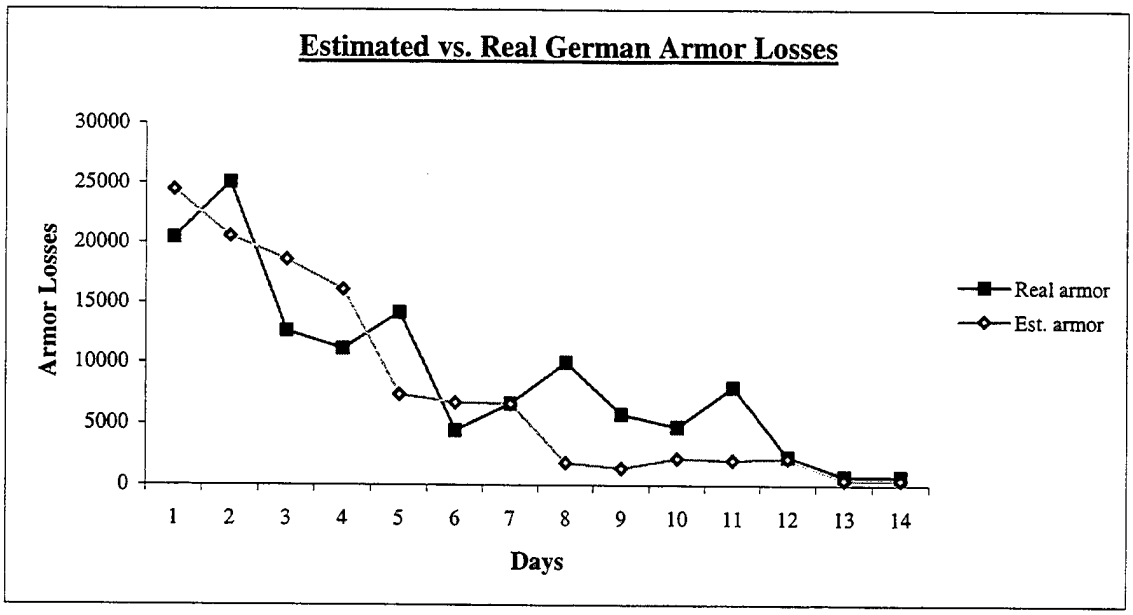

Figure 4.26. Estimated Versus Real German Armor Losses in the ACUD Data. The general pattern of the model is good.

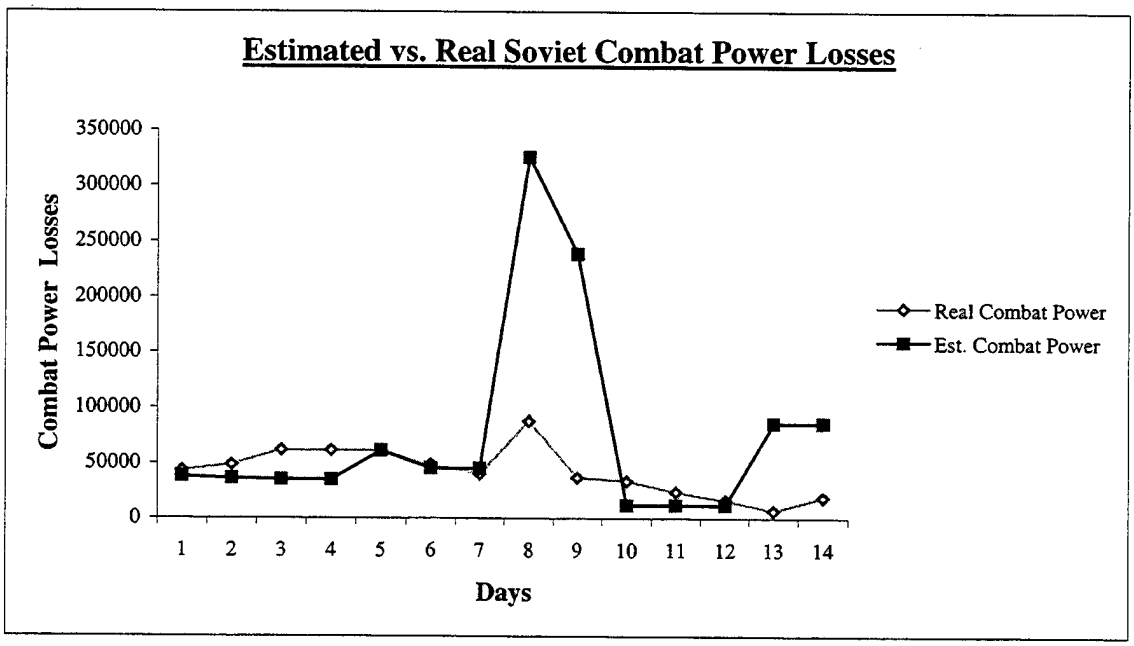

Figure 4.27. Estimated Versus Real Soviet Combat Power Losses in ACUD Data. The model overestimates the battle on days 8, 9, 13 and 14. On the other days, on which the Soviets were the defender, the general pattern is very plausible.

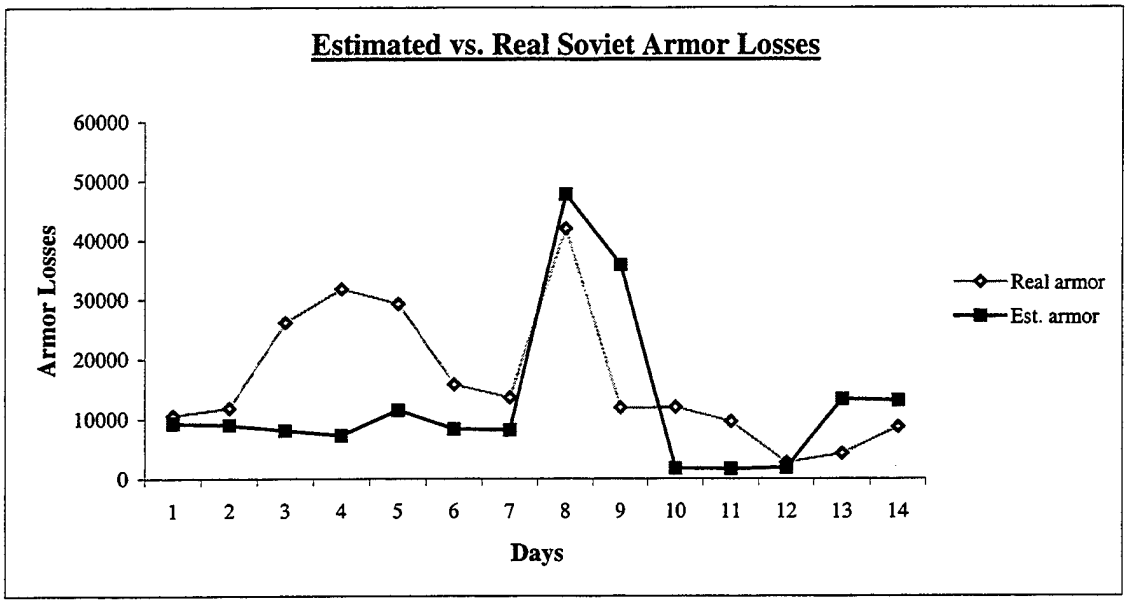

Figure 4.28. Estimated Versus Real Soviet Armor Losses in the ACUD Data. The model generally underestimates casualties, however it overestimates on days 8 and 9 when the Soviets attacked. Also, the model caught the spike in casualties.

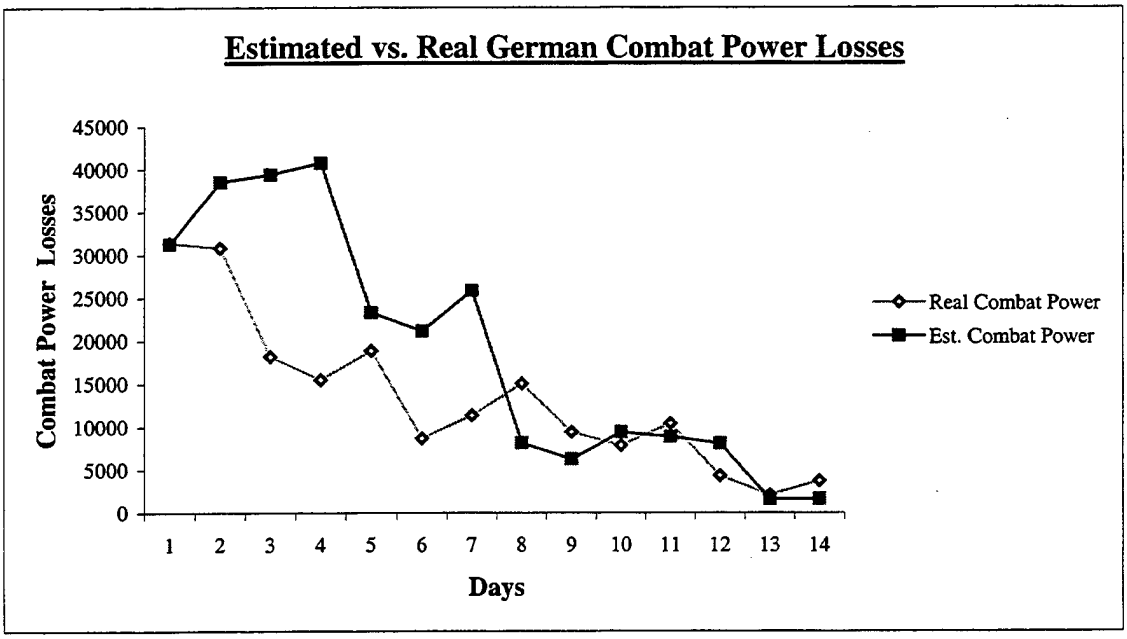

Figure 4.29. Estimated Versus Actual German Combat Power Losses in the CCUD Data. The model overestimates the battle on most days but as a whole, the pattern is not bad.

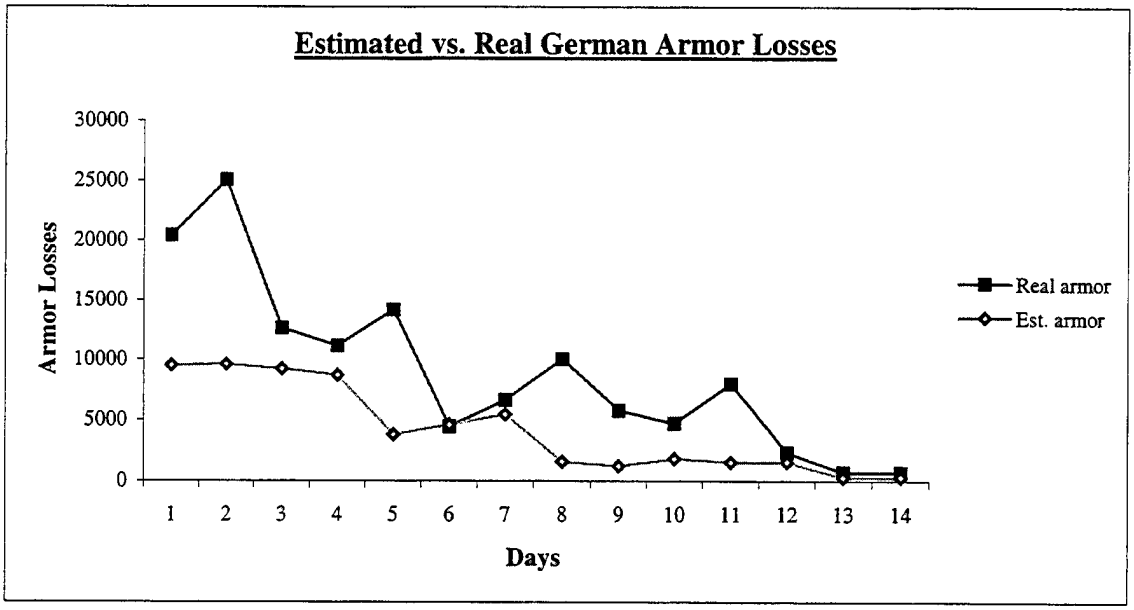

Figure 4.30. Estimated Versus Actual German Armor Losses in the CCUD Data. Although the model highly overestimates the casualties on the first two days, the general pattern of the model is good.

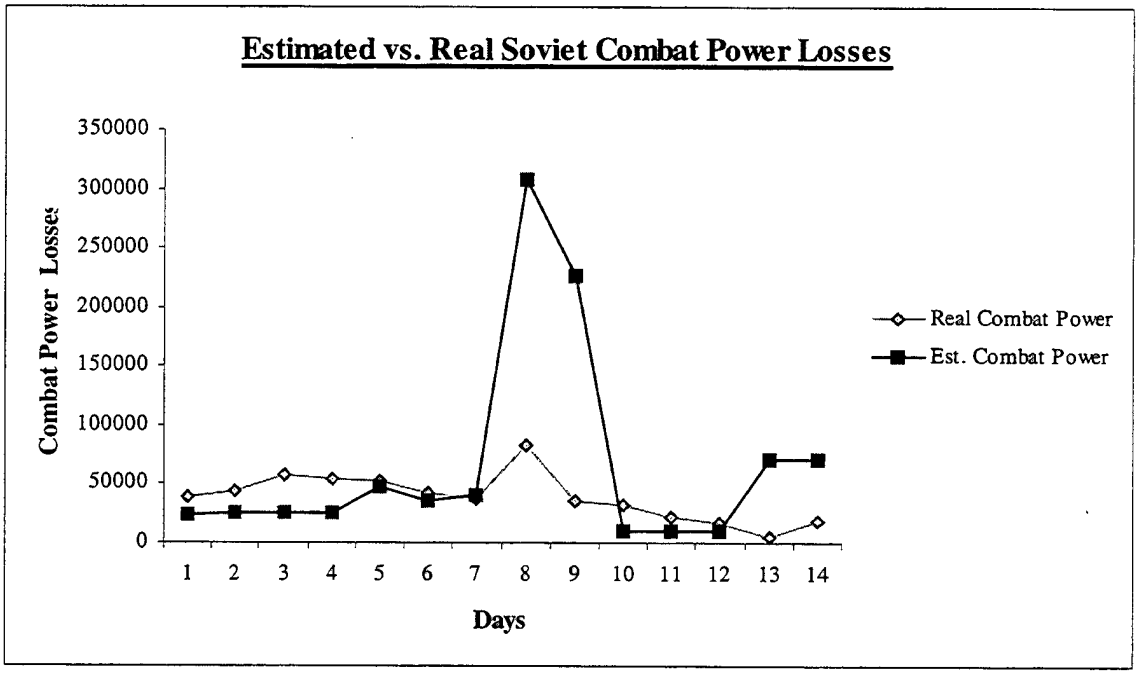

Figure 4.31 Estimated Versus Real Soviet Combat Power Losses in the CCUD Data. The model overestimates the battle on days 8, 9, 13 and 14. However, the model catches the trend.

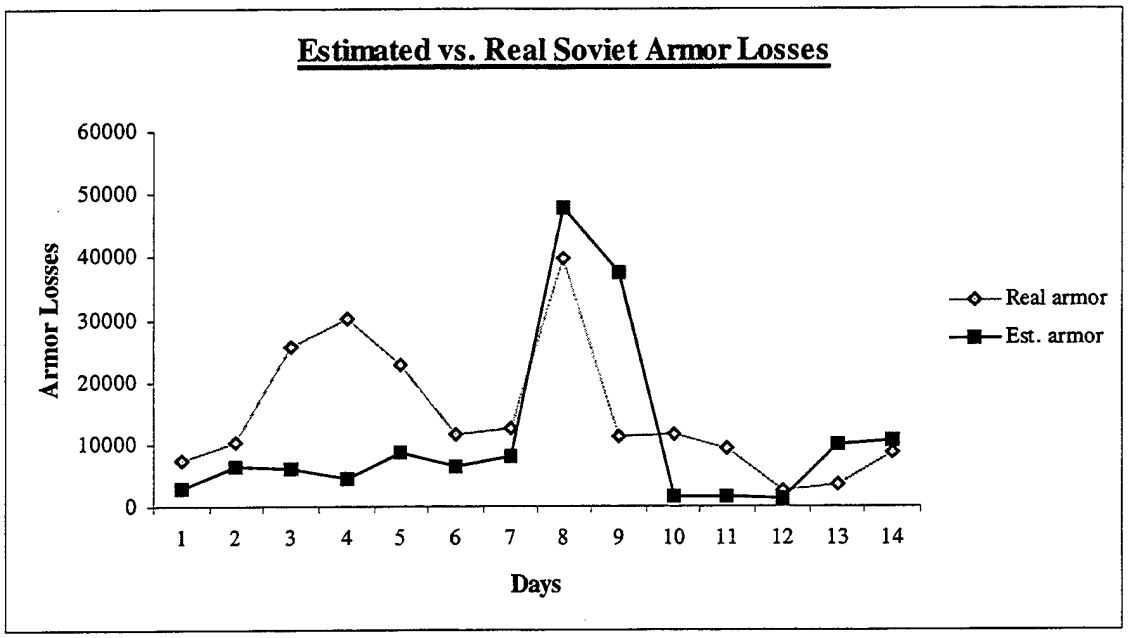

Figure 4.32. Estimated Versus Real Soviet Armor Losses in the CCUD Data. Although the model underestimates the casualties except for the days on which the Soviets attacked, the overall pattern is very plausible.

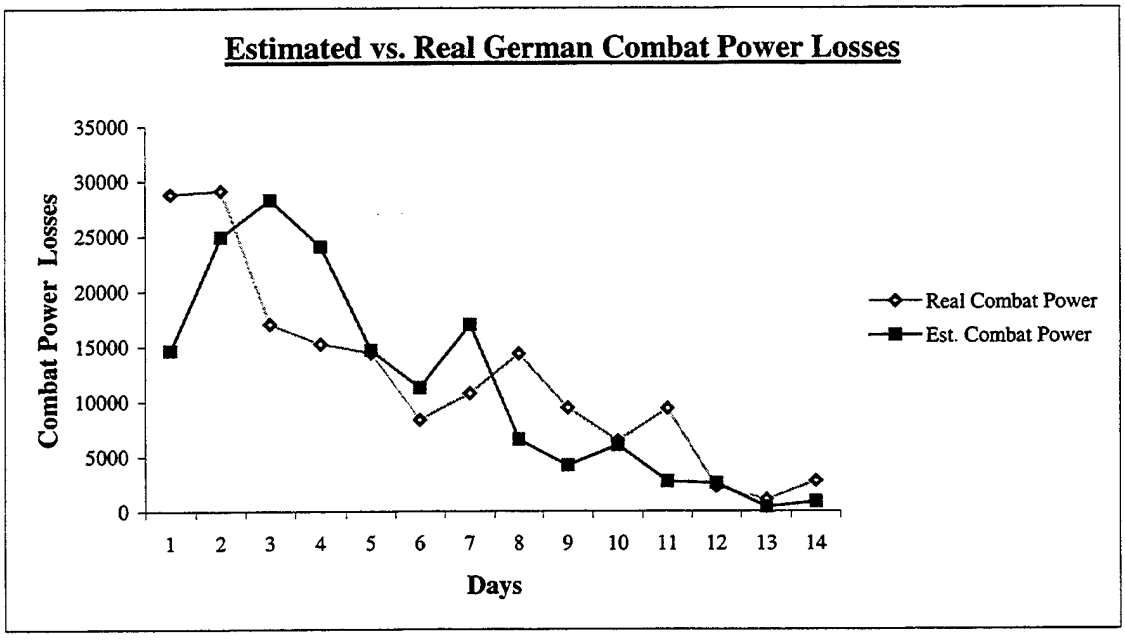

Figure 4.33. Estimated Versus Actual German Combat Power Losses in the FCUD Data. The general pattern of the model is good.

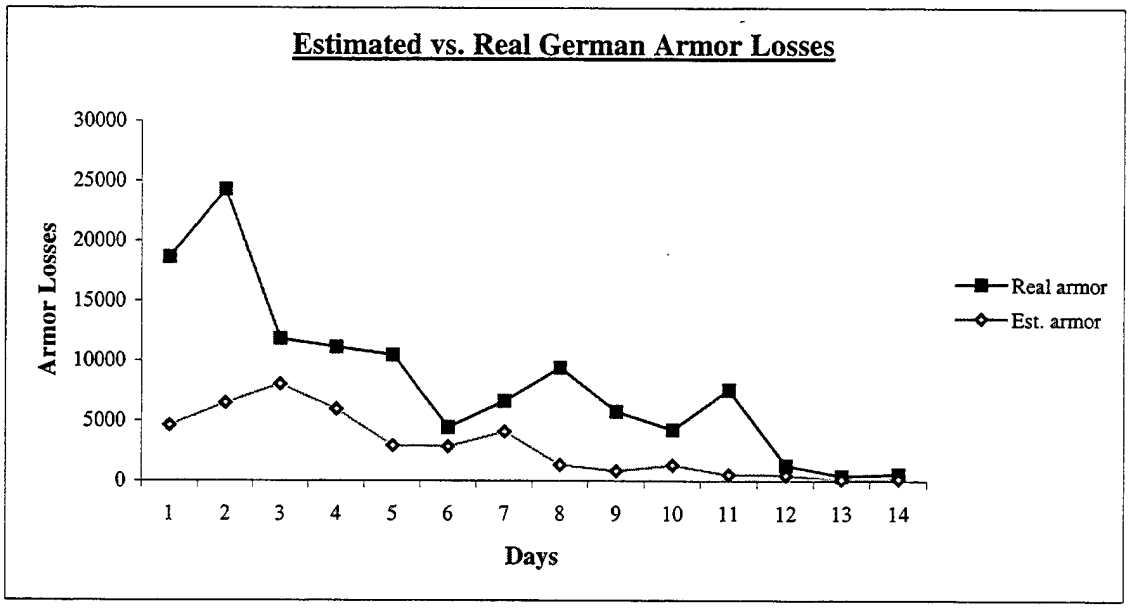

Figure 4.34. Estimated Versus Actual German Armor Losses in the FCUD Data. The model overestimates the battle except for the last three days.

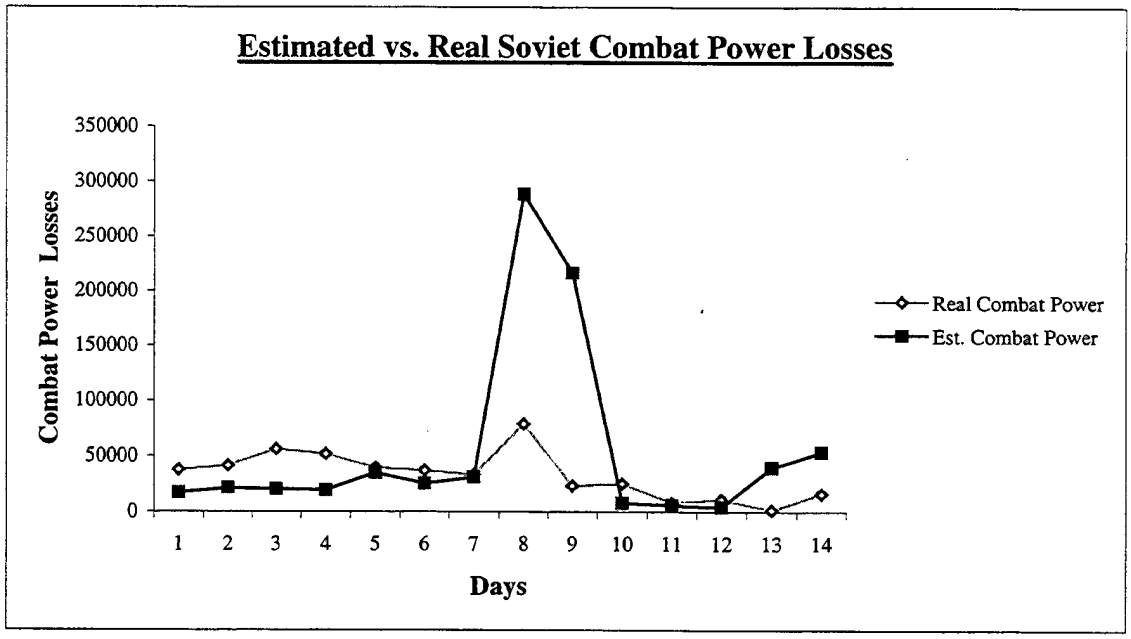

Figure 4.35. Estimated Versus Real Soviet Combat Power Losses in the FCUD Data. The model overestimates the battle on days 8, 9, 13 and 14. On the other days, the overall pattern is good.

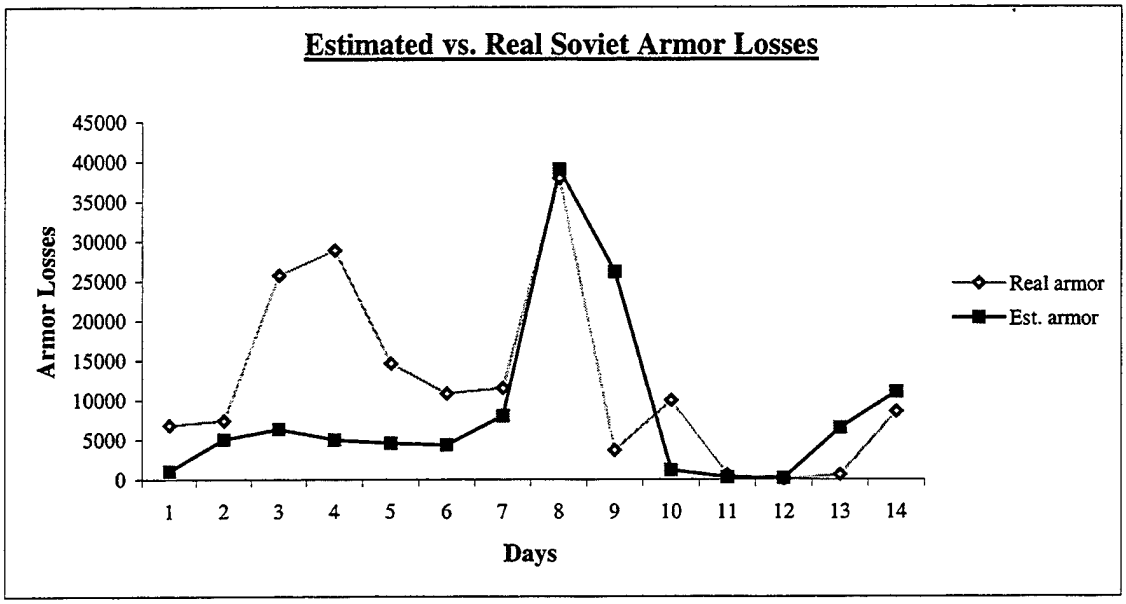

Figure 4.36. Estimated Versus Real Soviet Armor Losses in the FCUD Data. The model fits well on the 8th day of the battle. Although the model underestimates the casualties during the first days of the battle, the general pattern is good.

6. FLOT Movement Rates

As mentioned in Section 4, force ratio is also used to determine the FLOT movement rates in RAND's model. In RAND's SFS study, the FLOT movement rate is defined as a function of the type of battle and the relative attrition rates [Ref. 7:p42]. The equations presented in the SFS method [Ref. 7:p.42] are used to calculate the estimated FLOT movement rates. The only multiplier determined in the equations is FMR-Intens [Ref. 7:p.42]. The FMR-Intens is a measure of the attacker's priority for movement in the particular sector and not a measure of the overall battle intensity [Ref. 7:p.42]. The FMR-Intens multiplier was assigned as follows: for the second day of the battle (5 July), it is medium. For the 3rd, 4th and 5th days it is high and for the rest of the days it is low.

The KOSAVE study reports the average German progress in the northbound advance of the campaign [Ref. 14:p.8-1]. In this study, it is stated that "northbound advance is a suitable measure of progress because the primary German objective was a northward advance toward Kursk." The average German progress is given in Table 4.41 [Ref. 14:Data98-7-worksheet Chapt8].

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Average Progress (in km)	1.42	2.52	2.99	3.18	0.69	2.93	0.80	1.95	2.51	0.21	0.99	0.91	-0.24	0.34

Table 4.41. Daily Average German Northbound Progress Increased Every Day Except for the 13th Day (17 July).

Since the FLOT movement rate is always in the direction of the attack, it is not possible to compare the actual data to estimated rates on all days. Thus, the days in which the Germans are defined as the defender were skipped. Also, the first day of the battle is omitted, as in previous sections. Table 4.42 shows the days included in the model and the estimated FEBA movement rates for all three data sets.

Data	**Day**	**5 July**	**6**	**7**	**8**	**9**	**10**	**11**	**14**	**15**	**16**
	Real FMR	1.42	2.52	2.99	3.18	0.69	2.93	0.80	0.21	0.99	0.91
ACUD	**Est. FMR**	2.29	3.30	3.42	3.80	1.35	1.24	1.29	1.06	1.05	1.07
CCUD	**Est. FMR**	10.76	11.49	10.68	9.95	2.94	2.18	1.77	1.44	1.60	1.88
FCUD	**Est. FMR**	36.20	22.09	12.93	18.41	4.47	4.80	2.82	1.90	7.97	3.22

Table 4.42. Estimated and Real FMR for the Germans. FMR denotes the FLOT movement rate. The days are the actual dates of the battle.

The R^2 values of the estimated FLOT movement rates are 0.45, -24.10, and –177.20 for the ACUD, CCUD, and FCUD data sets respectively. In other words, it fits all combat units data fairly well, but other data sets very poorly.

Figures 4.37, 4.38 and 4.39 show the estimated versus real German FLOT movement rates for the days the German attacked for the ACUD, CCUD, and FCUD data sets respectively. The model fits better in the ACUD data set. Also, its signed rank test p-value indicates a good fit for the ACUD data set. The R^2 value for the ACUD data set is 0.45, which is very plausible for combat models. The only poor fit is observed on the 6th day of the battle. However, the general pattern is good for the ACUD data set.

The model fits poorly in the CCUD and FCUD data sets. This fact is also supported by their R^2 values and very small signed rank test p-values.

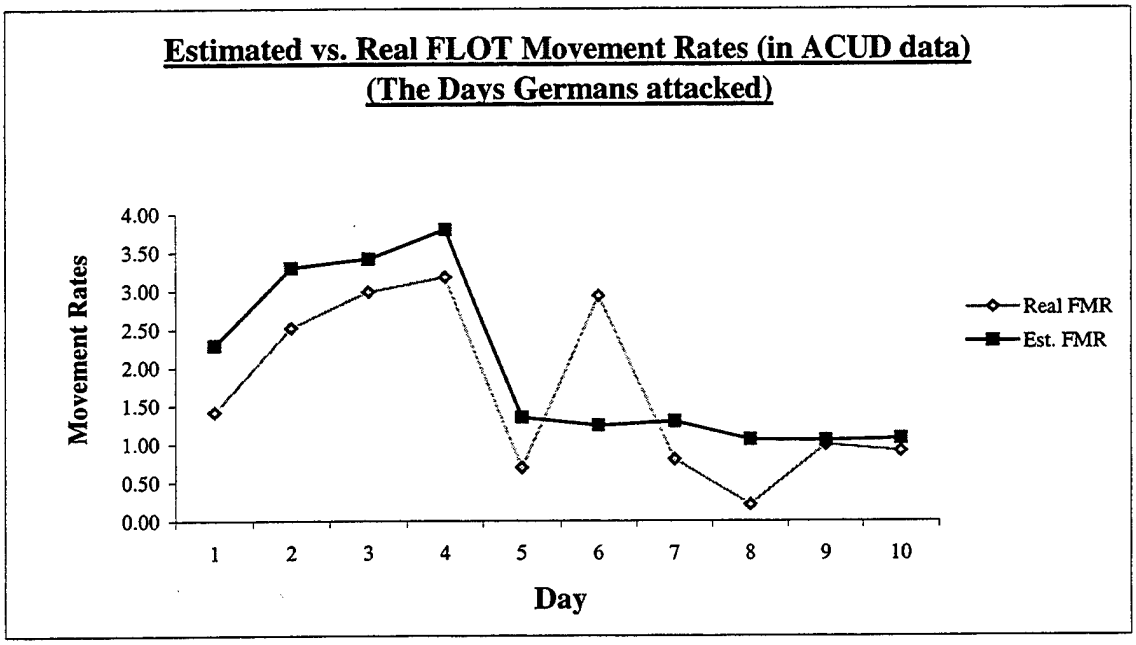

Figure 4.37. Estimated Versus Real FMR for the Germans in the ACUD Data Set. The model fits fairly well except for the 6th day.

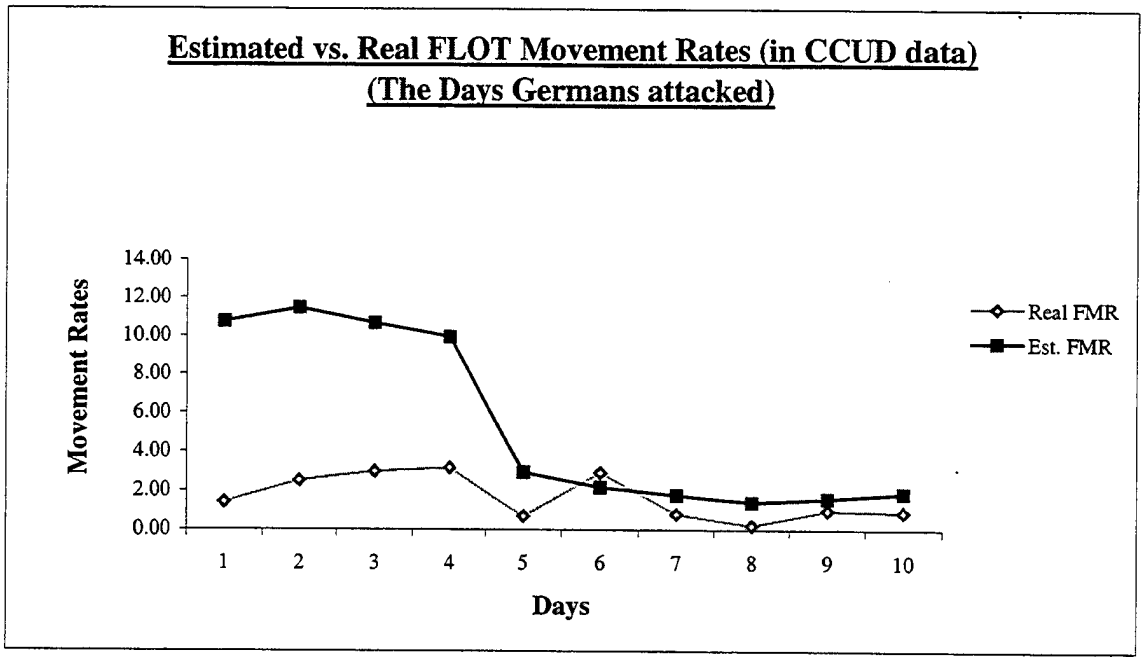

Figure 4.38. Estimated Versus Real FMR for the Germans in the CCUD Data Set. The model highly overestimates the battle during the first four days. However, towards the end, the fit is fairly good.

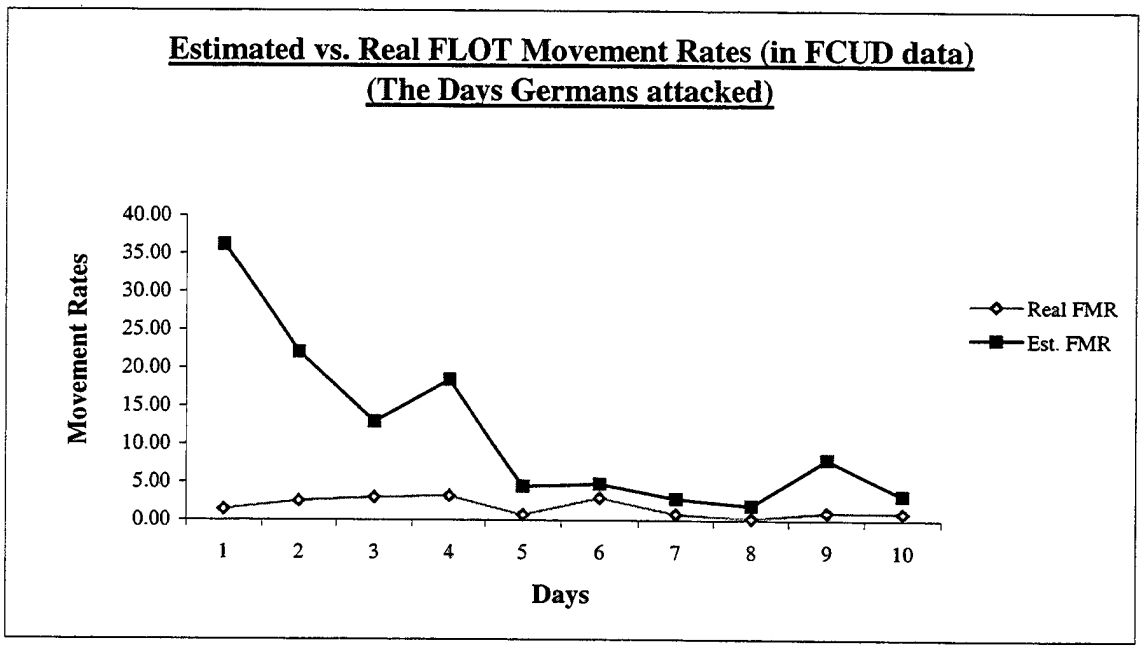

Figure 4.39. Estimated Versus Real FMR for the Germans in the FCUD Data Set. The model overestimates the battle for each day. Again, during the first days, the overestimation is very high.

C. APPLICATION OF THE QUANTIFIED JUDGMENT MODEL (QJM) METHODOLOGY

As a result of many years of research and effort, Colonel Trevor Dupuy, US Army Retired, invented a new and revolutionary approach to the study of warfare [Ref. 22:p.xvii]. The approach is called the Quantified Judgment Method of Analysis (QJMA) and the resulting model or simulation of combat, Quantified Judgment Model (QJM).

The basis for the QJM approach towards attrition is the experience derived from the analysis of attrition in modern warfare [Ref. 23:p.181]. In the QJM model, the basic attrition calculation is for personnel losses. Losses for other weapon types and materials are based on their historical relationship to personnel losses. The basic relationship to determine the personnel loss rates is as follows [Ref. 23:p.181]:

Personnel Loss Rate = (standard casualty rate) * (variable factors)

The standard casualty rate is the average casualty rate experienced for a particular war or historical era. This value may be determined by historical analysis, or assumed for future combat [Ref. 23:p.181]. For the details of the standard casualty rate, see [Ref. 23:p.181] and [Ref. 3:p.105].

In this section, the QJM model is applied to the three data sets presented in the earlier section. A similar format for presenting QJM as seen in the application of RAND's algorithm will be followed. The basic equations will be provided, an explanation for determining the variables factors will be given, the model will be applied to the data and the results will be presented in figures.

1. Personnel Attrition

As mentioned above, the model starts with determining personnel losses. The personnel casualty equation is as follows:

$$C = .04(N * rc * hc * uc * tz * op * Su) * so \quad (4.7)$$

where:

C: daily casualties incurred by then force

N: Personnel strength of the force

Rc: Terrain factor for casualties

Hc: Weather factor for casualties

Uc: Posture factor for casualties

Tz: Strength-size factor

Op: opposition factor

Su: Surprise factor

So: Sophistication factor

Lookup tables determine these factors. For more explanation of these factors, see [Ref. 3:p.105-106]. Possibly the most critical of the factors in Equation 4.7 is the opposition factor. This is based upon the relative combat power of the opposing forces [Ref. 3:p. 107]. In order to solve Equation 4.7, it is first necessary to determine the opposition factor. It is also defined from lookup-tables, but it is then necessary to compute the combat power ratio for each side. Combat power is represented by a simple equation:

$$P = S * V * CEV \quad (4.8)$$

Where:

P: Combat power of a force

S: Force Strength (total firepower value, modified by circumstances of combat)

V: Variable factors applicable to combat circumstances

CEV: Relative Combat Effectiveness Value (with respect to the opponent's effectiveness)

Equation 4.8 can be expanded as:

$$P = (N * so) * (m * u * r * h * Su) * (CEV) \quad (4.9)$$

Where:

N: Personnel strength of a force

So: Weapon firepower sophistication/efficiency

M: Mobility factor

U: Posture factor

R: terrain factor

H: weather factor

Su: surprise factor

CEV: military effectiveness relative to opponent

Once Equation 4.9 is computed for both forces, the combat power ratio is calculated. Then, using this ratio, the opposition factor is determined from the lookup tables. After having determined the opposition factor, it is put into Equation 4.7 to solve for the daily personnel casualties. Tables 4.43 and 4.44 show the values of the factors

used in this study, and the combat power ratios for the Germans and Soviets respectively in all three data sets.

Day	So	m	u	r	h	Su	CEV	Pg/Ps		
								ACUD	CCUD	FCUD
1	1.3	1.2	1	1	1	1	3	**1.98**	**4.83**	**9.77**
2	1.3	1.2	1	1	1	1	3	**1.99**	**4.17**	**6.17**
3	1.3	1.2	1	1	1	1	3	**2.01**	**3.83**	**4.32**
4	1.3	1.2	1	1	1	1	3	**2.13**	**3.59**	**5.21**
5	1.3	1.2	1	1	1	1	3	**2.52**	**3.95**	**4.88**
6	1.3	1.2	1	1	1	1	3	**2.56**	**3.26**	**4.70**
7	1.3	1.2	1	1	1	1	3	**2.58**	**3.05**	**4.14**
8	1.3	1.2	1.4	1	1	1	3	**4.51**	**5.31**	**6.81**
9	1.3	1.2	1.4	1	1	1	3	**4.63**	**5.40**	**10.46**
10	1.3	1.2	1	1	1	1	3	**2.79**	**3.27**	**3.94**
11	1.3	1.2	1	1	1	1	3	**2.75**	**3.76**	**9.81**
12	1.3	1.2	1	1	1	1	3	**2.77**	**4.15**	**5.86**
13	1.3	1.2	1.4	1	1	1	3	**4.67**	**5.89**	**25.80**
14	1.3	1.2	1.4	1	1	1	3	**4.69**	**5.46**	**10.33**

Table 4.43. Daily the German Combat Power and Factors Used in its Calculation. The last three columns show the German combat power ratio for all three data sets. The Pg and Ps denote the combat power of the Germans and Soviets respectively.

Day	So	m	u	r	h	Su	CEV	Pg/Ps		
								ACUD	CCUD	FCUD
1	1	1	1.4	1	1	1	1	**0.50**	**0.21**	**0.10**
2	1	1	1.4	1	1	1	1	**0.50**	**0.24**	**0.16**
3	1	1	1.4	1	1	1	1	**0.50**	**0.26**	**0.23**
4	1	1	1.4	1	1	1	1	**0.47**	**0.28**	**0.19**
5	1	1	1.2	1	1	1	1	**0.40**	**0.25**	**0.20**
6	1	1	1.2	1	1	1	1	**0.39**	**0.31**	**0.21**
7	1	1	1.2	1	1	1	1	**0.39**	**0.33**	**0.24**
8	1	1	1	1	1	1	1	**0.22**	**0.19**	**0.15**
9	1	1	1	1	1	1	1	**0.22**	**0.19**	**0.10**
10	1	1	1.2	1	1	1	1	**0.36**	**0.31**	**0.25**
11	1	1	1.2	1	1	1	1	**0.36**	**0.27**	**0.10**
12	1	1	1.2	1	1	1	1	**0.36**	**0.24**	**0.17**
13	1	1	1	1	1	1	1	**0.21**	**0.17**	**0.04**
14	1	1	1	1	1	1	1	**0.21**	**0.18**	**0.10**

Table 4.44. Daily the Soviet Combat Power and Factors Used in its Calculation. The last three columns show the Soviet combat power ratio for all three data sets. The Pg and Ps denote the combat power of the Germans and Soviets respectively.

Despite their lower personnel strength, the Germans have a higher combat power ratio over the Soviets for each day in the ACUD data set. This reflects the effects of the factors used in Equation 4.9. For all data sets, the German ratio is greater than the Soviet's by a power of at least four. When the data sets are compared, it is clear that the FCUD data set gives the highest combat power ratio in favor of the Germans. This can be explained with the engagement percentages of the forces. While an average of 92 percentage of the German personnel strength is in contact with the enemy forces, the Soviets had an average of 67 percent personnel strength in contact. The following paragraphs explain how the factors were determined.

The purpose of the sophistication factor is to reflect approximately one or more of the following [Ref. 3:p.152]:

- More sophisticated weapons
- A higher proportion of crew served weapons, particularly artillery
- A higher proportion of close air support and battlefield interdiction

Since the Germans had higher close air support and slightly more sophisticated weapons than the Soviets, the sophistication factor (so) is determined as 1.3 in favor of the Germans.

The mobility factor is defined as 1 for both sides. The posture factor is determined as in previous sections. Due to the difficulties explained in Section B, the terrain factor is defined as 1, which does not affect the equations. Also, the weather factor was not used, i.e., it is set as 1. The surprise factor (Su) is not used in the equations as both sides had months of preparation. The CEV value of 3 was used in favor of the Germans. The reason for this, as explained in Section B, is that on the Eastern Front, the German combat

effectiveness superiority varied in both World Wars between 2.0 and 3.0 [Ref. 3:p.108]. Although the upper limit (3) based on Turkes's research was selected [Ref. 11], further analysis is required.

After having calculated combat power ratios for each side, the next issue is determining the opposition factor from Table M [Ref. 3:p.150]. The opposition factor is put into Equation 4.7 with other factors, and personnel casualties are computed. Tables 4.45 and 4.46 present the factors used in Equation 4.7 for all data sets for the Germans and Soviets respectively. Since the opposition factor depends on the combat power ratio, it is given for all data sets. The next paragraph explains how factors are defined.

The terrain, posture, weather and surprise factors are defined the same as in the computation for Equation 4.9. The sophistication factor is 1, which does not affect the casualties for both forces. The opposition factor is defined from Table M [Ref. 3:p.150] using combat power ratios. The strength size factor recognizes that the smaller the force, the higher the casualty rate. It is normalized to the size of a typical World War II division, or 15,000 men, as shown in Table L [Ref. 3:p.106-149]. Since our data is not at the division level, and instead is aggregated for the whole sector, the smallest value for this factor is used which is 0.3 for over 100,000 men. After daily personnel losses have been calculated, Equation 4.10 computes the casualty rate, in percent for that day.

$$CR = C / N \tag{4.10}$$

Where:

CR: casualty rate

C: daily casualties (from Equation 4.7)

N: numerical strength of the force

Day	rc	Hc	Uc	tz	Op			Su	so
					ACUD	CCUD	FCUD		
1	1	1	1	0.3	0.75	0.5	0.4	1	1
2	1	1	1	0.3	0.75	0.55	0.4	1	1
3	1	1	1	0.3	0.75	0.6	0.55	1	1
4	1	1	1	0.3	0.75	0.6	0.45	1	1
5	1	1	1	0.3	0.7	0.6	0.5	1	1
6	1	1	1	0.3	0.65	0.6	0.5	1	1
7	1	1	1	0.3	0.65	0.6	0.55	1	1
8	1	1	0.85	0.3	0.65	0.45	0.4	1	1
9	1	1	0.85	0.3	0.5	0.45	0.4	1	1
10	1	1	1	0.3	0.65	0.6	0.6	1	1
11	1	1	1	0.3	0.65	0.6	0.4	1	1
12	1	1	1	0.3	0.65	0.55	0.45	1	1
13	1	1	0.85	0.3	0.5	0.45	0.4	1	1
14	1	1	0.85	0.3	0.5	0.45	0.4	1	1

Table 4.45. Daily the German Factors used in Equation 4.7.

Day	Rc	hc	uc	tz	Op			Su	so
					ACUD	CCUD	FCUD		
1	1	1	0.85	0.3	1.35	1.7	2.3	1	1
2	1	1	0.85	0.3	1.35	1.7	1.9	1	1
3	1	1	0.85	0.3	1.35	1.6	1.7	1	1
4	1	1	0.85	0.3	1.35	1.6	1.8	1	1
5	1	1	0.9	0.3	1.45	1.6	1.7	1	1
6	1	1	0.9	0.3	1.45	1.5	1.7	1	1
7	1	1	0.9	0.3	1.45	1.5	1.7	1	1
8	1	1	1	0.3	1.7	1.8	2	1	1
9	1	1	1	0.3	1.7	1.8	2.5	1	1
10	1	1	0.9	0.3	1.45	1.5	1.6	1	1
11	1	1	0.9	0.3	1.45	1.6	2.3	1	1
12	1	1	0.9	0.3	1.45	1.7	1.8	1	1
13	1	1	1	0.3	1.7	1.9	2.5	1	1
14	1	1	1	0.3	1.7	1.8	2.5	1	1

Table 4.46. Daily the Soviet Factors Used in Equation 4.7.

This rate is used in the computation of material losses, which will be explained in the following section.

2. Weapon Losses

The weapon losses are calculated based on personnel casualty rates computed in the previous section. There are two equations in the QJM model to calculate the tank and artillery losses. These equations will be presented in the following sections. Other weapon losses are calculated according to personnel, tank and artillery losses. These are:

- APC: same as tank loss rate
- Antitank weapons: same as personnel casualty rate (CR)
- Infantry heavy weapons: 1.5 * CR
- Air defense Artillery: same as artillery losses

a. Tank Loss Rate

The simple equation for tank losses is [Ref. 3:p.112]:

$$DTLa = CR * CKT * NT * CEVd * tz * uc * Sui \qquad (4.11)$$

Where:

DTL: Daily tank loss

a: Attacker identifier

d: Defender identifier

CR: Personnel casualty loss

CKT: Standard tank loss rate; for the attacker this is 6; for the defender it is 3

NT: Number of tanks

CEVo: Opponent's relative combat effectiveness

tzi: Strength-size factor for tanks

Sui: Surprise attrition factor for the tanks

The CEV factor is defined as 3 in favor of the Germans. The strength-size factor for tanks is determined from Table L [Ref. 3:p.149] using the daily number of each side. The surprise attrition factor for tanks is defined as 1. The CKT value is defined according to combat posture except for 12 July. This is the day that a German breakthrough attempt resulted in a major close quarters tank battle near the town of Prokhorovka [Ref. 14:pp.2-3]. Thus, on this particular day, the CKT value of 6 is used that shows both sides in an attacking position. These factors are presented in Tables 4.37 and 4.38.

b. Artillery Loss Rate

Artillery loss rates can be calculated by the following equation [Ref. 3:p.113]:

$$DALa = CR * CKA * NA * CEVd \qquad (4.12)$$

Where:

DAL: Daily artillery loss in weapons

a: Attacker identifier

d: Defender identifier

CR: Personnel casualty rate

CKA: Standard artillery loss rate factor which is 0.30 for self-propelled weapons and 0.1 for towed weapons

NA: Number of artillery weapons

CEV: Relative combat effectiveness

The same CEV value of 3 was used again. Since the artillery was not partitioned as self-propelled or towed, the average value of 2 was used for the CKA value in this study. Table 4.47 shows the factors used in Equations 4.11 and 4.12 for all data sets. Since the tank strength size factor depends upon the number of tanks, it is determined for each data set.

Day	CKT (German)	CKT (Soviet)	CEV (German)	tzi (German)			tzi (Soviet)			CKA
				ACUD	CCUD	FCUD	ACUD	CCUD	FCUD	
1	12	6	3	0.86	0.86	0.86	0.8	0.91	1.2	0.2
2	12	6	3	0.86	0.86	0.86	0.8	0.83	0.86	0.2
3	12	6	3	0.86	0.86	0.86	0.8	0.86	0.86	0.2
4	12	6	3	0.89	0.89	0.89	0.83	0.86	0.86	0.2
5	12	6	3	0.9	0.91	0.91	0.83	0.86	0.92	0.2
6	12	6	3	0.89	0.89	0.89	0.83	0.86	0.9	0.2
7	12	6	3	0.89	0.89	0.89	0.83	0.83	0.9	0.2
8	12	12	3	0.9	0.9	0.9	0.86	0.86	0.89	0.2
9	6	12	3	0.9	0.9	0.9	0.86	0.86	0.93	0.2
10	12	6	3	0.9	0.9	0.9	0.86	0.86	0.89	0.2
11	12	6	3	0.9	0.9	0.92	0.86	0.86	1.3	0.2
12	12	6	3	0.89	0.9	1	0.86	0.89	1.3	0.2
13	6	12	3	0.89	0.9	0.92	0.86	0.89	1	0.2
14	6	12	3	0.89	0.9	0.92	0.86	0.86	0.9	0.2

Table 4.47. The Values of the Factors Used in Equations 4.11 and 4.12 for all of the Data Sets.

3. Results

In this section, all the results are presented for the QJM model. Again, the R^2 value and the comparison figures were used to show the quality of the fits. The personnel, tank, artillery, APC, ATH, ATL, AA, and combat power losses of the Germans and Soviets for all three data sets are computed and their R^2 values are presented in Table

4.48. The non-parametric Wilcoxon signed-rank test to test the hypothesis of whether or not the difference between actual and estimated losses is the same is also used in this method and presented in Table 4.49. The combat power represents only these weapon types with the same firepower score values used in the previous sections.

Data Set	Forces	Personnel	Tank	ARTY	APC	ATH	MTR	ATL	AA	Combat Power
ACUD	GERMAN	**0.31**	-0.02	-1.74	-39.82	-1.81	-0.13	-1.31	-2.05	**0.20**
	SOVIET	-27.72	-3.40	-119.52	-101.06	-336.34	-33.93	-198.77	-91.25	-2.49
CCUD	GERMAN	-0.19	-0.29	-1.97	-23.52	-1.82	-0.13	-1.85	-2.17	-0.25
	SOVIET	-15.80	-1.23	-87.98	-66.41	-120.15	-16.34	-100.73	-13.67	-1.07
FCUD	GERMAN	-0.33	-0.48	-1.67	-11.54	-1.48	-0.01	-1.69	-1.96	-0.46
	SOVIET	-14.61	-0.17	-87.48	-66.95	-83.99	-15.06	-76.55	-7.06	-0.76

Table 4.48. The R^2 Values for both Sides in the Three Data Sets.

Data Set	Forces	Personnel	Tank	ARTY	APC	ATH	MTR	ATL	AA	Combat Power
ACUD	GERMAN	**0.95**	0.02	0.0001	1e-4	1e-4	0.02	0.002	2e-4	0.04
	SOVIET	**0.5**	1e-4	**0.09**	**0.62**	0.003	**0.07**	0.002	0.11	0.02
CCUD	GERMAN	**0.11**	0.01	0.0001	1e-4	1e-4	**0.07**	6e-4	1e-4	0.01
	SOVIET	0.02	0.005	0.01	0.02	0.009	0.02	0.001	**0.2**	**0.9**
FCUD	GERMAN	0.01	0.002	1e-4	1e-4	1e-4	0.02	2e-4	2e-4	0.002
	SOVIET	0.002	0.04	0.005	0.01	4e-4	0.002	2e-4	**0.17**	**0.21**

Table 4.49. The p-Values from the Wilcoxon Signed-Rank Test for all of the Data Sets for both Sides. The significant values are highlighted.

The positive R^2 values for the German personnel and combat power losses occur. The German personnel R^2 value is the best for all models in this study. The QJM model fits better for the Germans than the Soviets in all data sets. When comparing the data sets, which are similar to the previous models, the Soviet R^2 values are better in the FCUD data set than the ones in the other two data sets. This suggests that the classification of the data improves the quality of the fits. Although, for the Germans, this result is not clear, overall the model fits plausibly for the FCUD data set.

The comparison figures will be presented to provide better insight into the quality of the fits. Again, the combat power, tank, and personnel losses are presented.

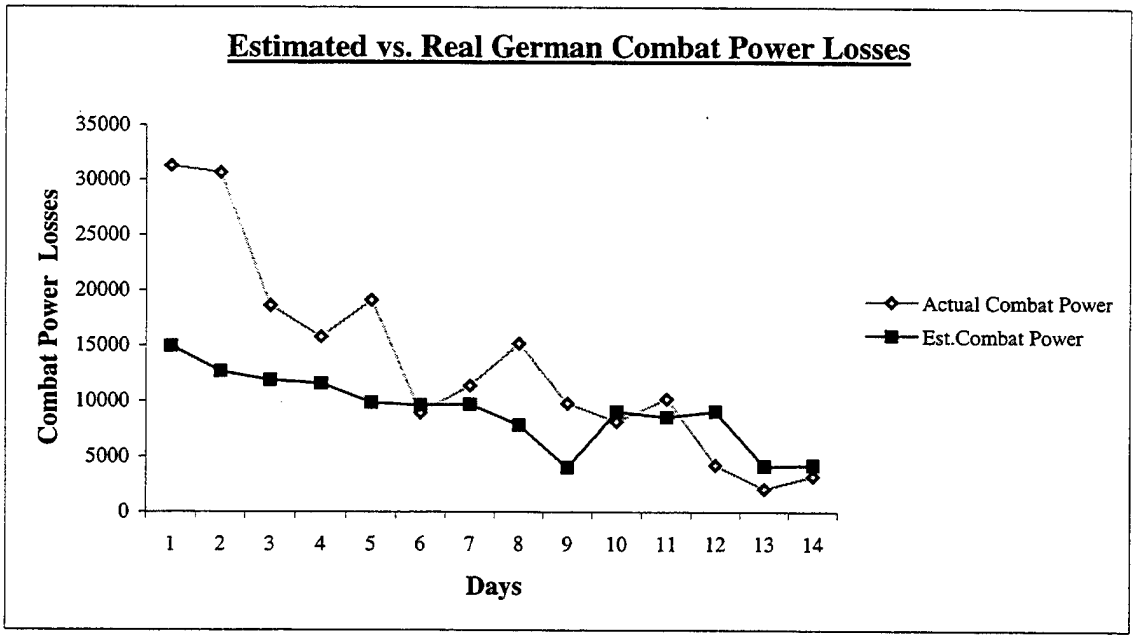

Figure 4.40. Estimated Versus Real German Combat Power Losses in the ACUD Data Set. The model underestimates the battle for the first 5 days.

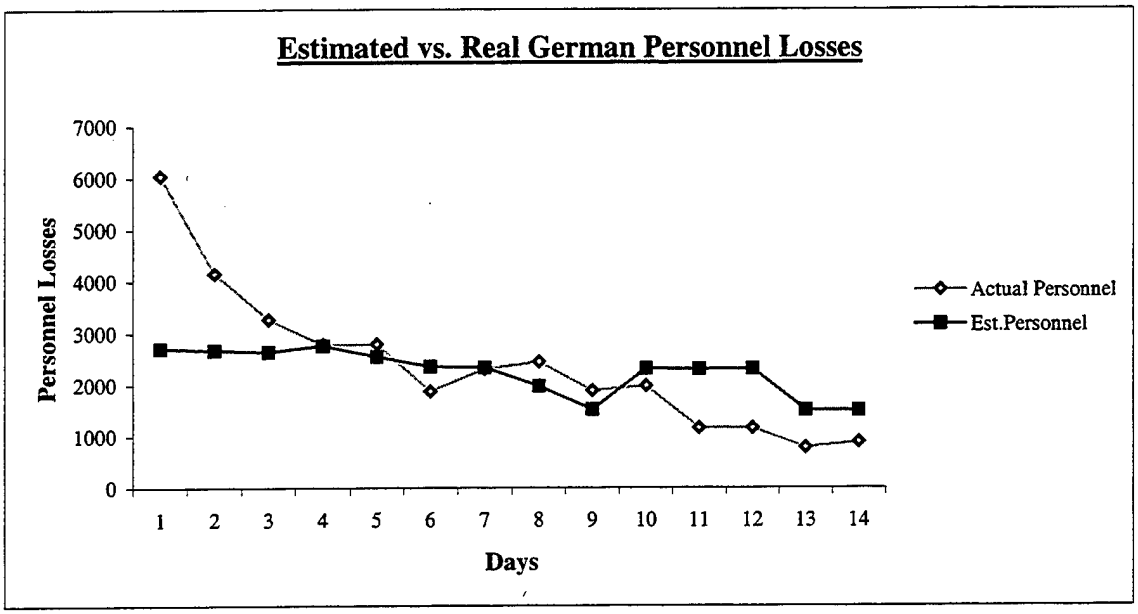

Figure 4.41. Estimated Versus Real German Personnel Losses in the ACUD Data Set. The model underestimates the first 3 days, and overestimates the last 4 days. There is not any significant outlier which is supported by its R^2 value. The general pattern of the model is fairly good.

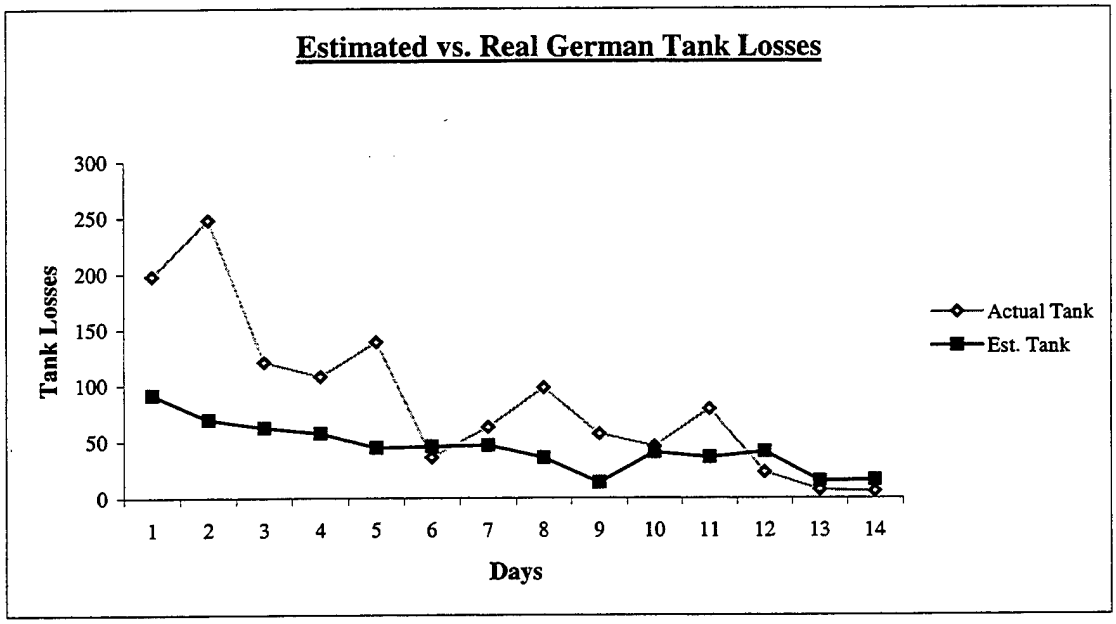

Figure 4.42. Estimated Versus Actual German Tank Losses in the ACUD Data Set. On most days, the model underestimates the battle. However, the general trend is good.

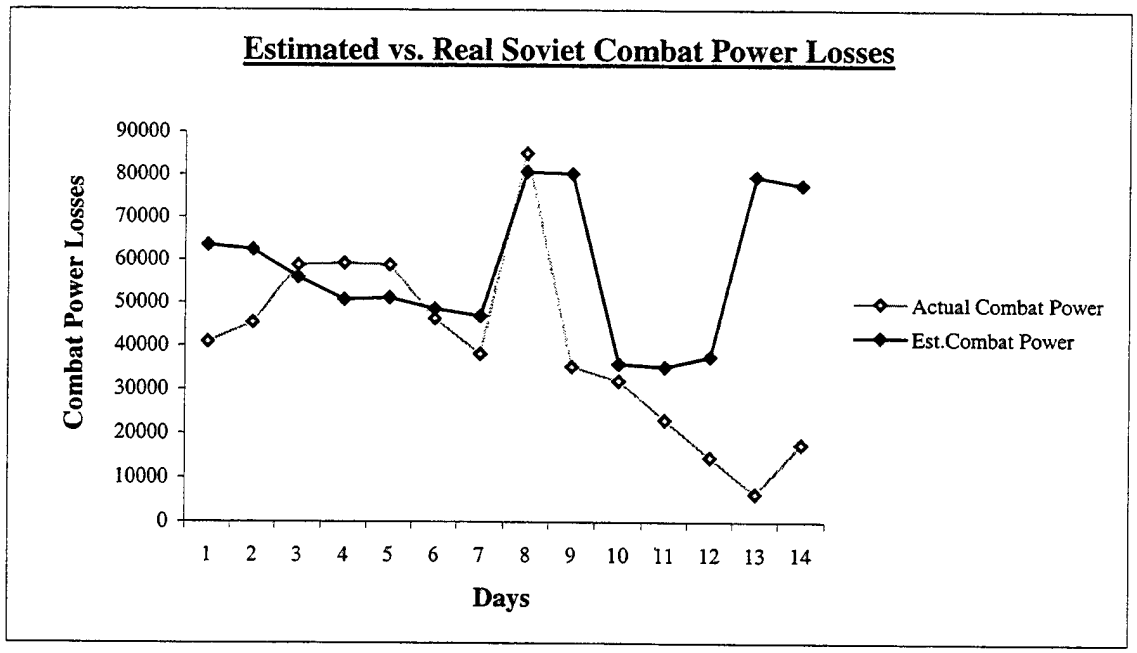

Figure 4.43. Estimated Versus Actual Soviet Combat Power Losses in the ACUD Data Set. Although, the model overestimates the battle on the 9th day and for the last two days, the general pattern is not bad.

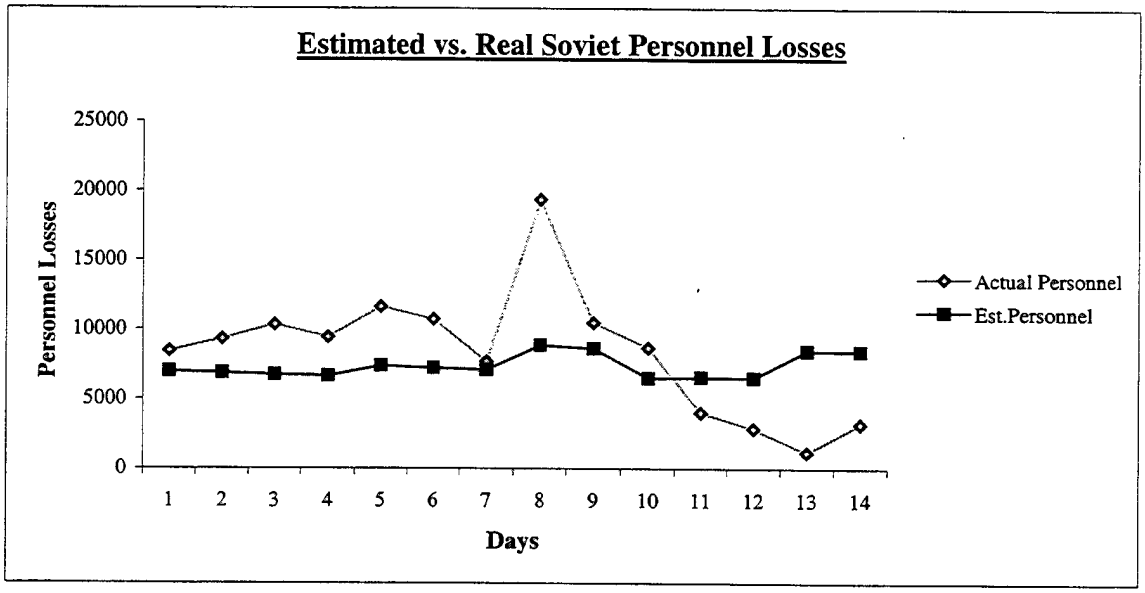

Figure 4.44. Estimated Versus Actual Soviet Personnel Losses in the ACUD Data Set. The model mostly underestimates the battle except for the last four days. Overall, the trend of the model is good.

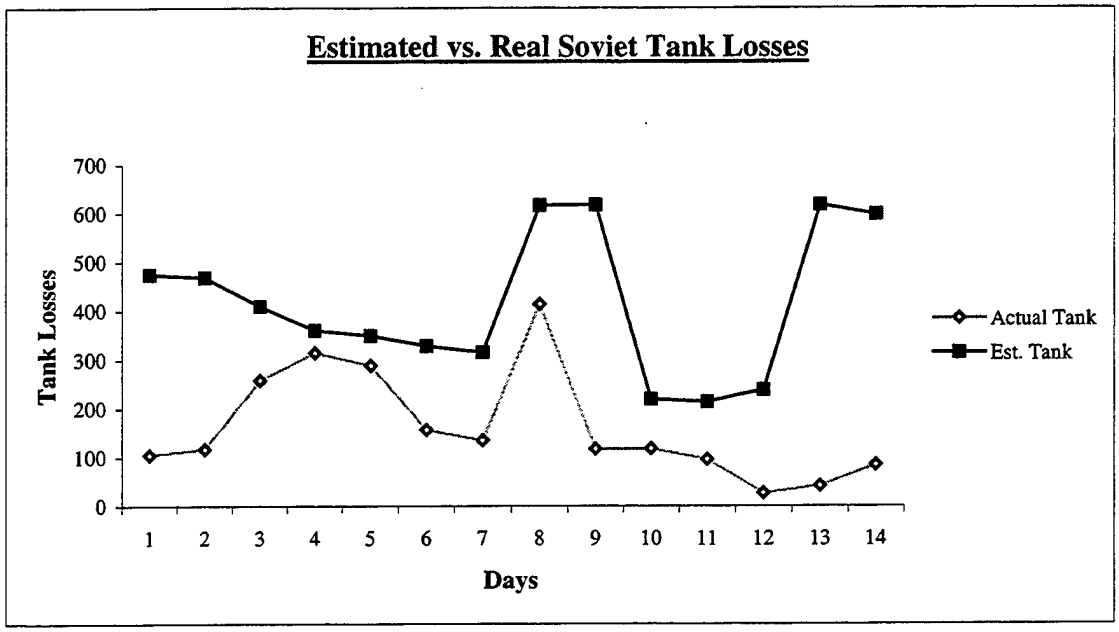

Figure 4.45. Estimated Versus Actual Soviet Tank Losses in the ACUD Data Set. The model overestimates the battle during the whole campaign. It has a similar pattern with the combat power figure.

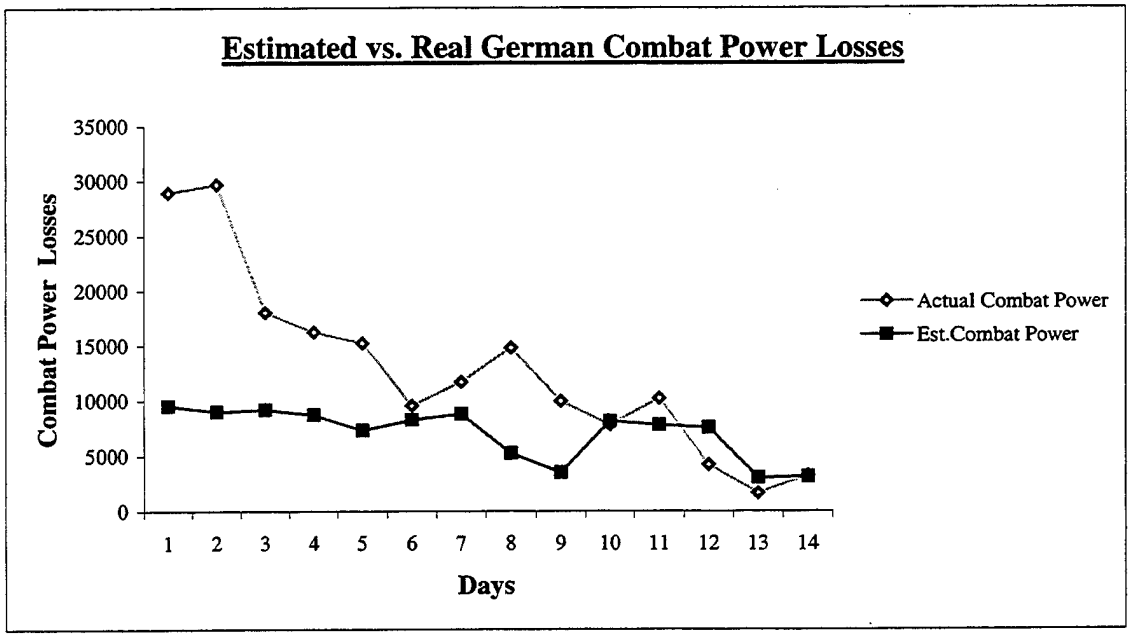

Figure 4.46. Estimated Versus Real German Combat Power Losses in the CCUD Data Set. The model underestimates the battle for the first 5 days. It has the same pattern as the one in the ACUD data set.

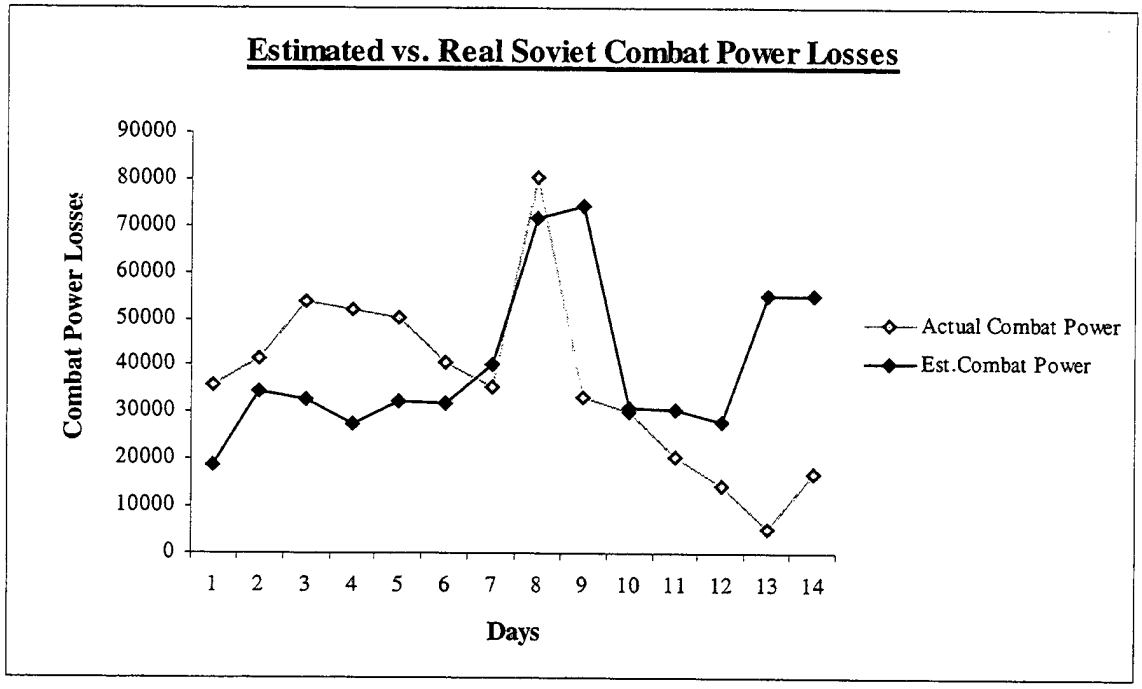

Figure 4.47. Estimated Versus Actual Soviet Combat Power Losses in the CCUD Data Set. The model overestimates the battle on the 9th day and for the last four days. Except for the last days, the general pattern is not bad.

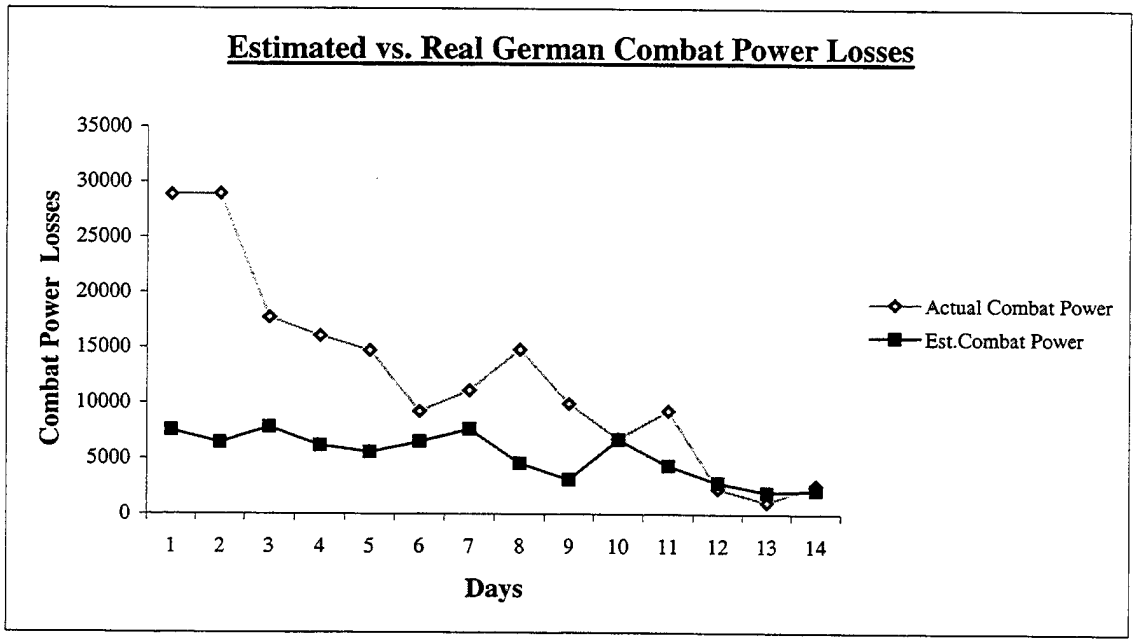

Figure 4.48. Estimated Versus Real German Combat Power Losses in the FCUD Data Set. The model mostly underestimates the battle, but it gives a good fit towards the end.

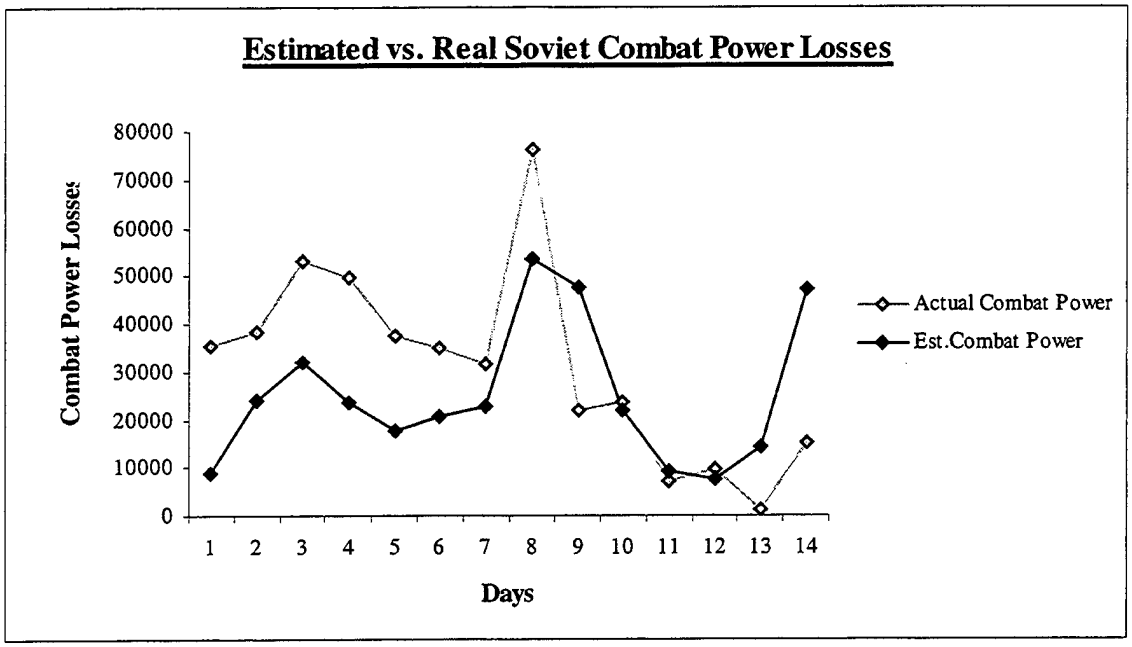

Figure 4.49. Estimated Versus Actual Soviet Combat Power Losses in the FCUD Data Set. The model overestimates the battle on the 9th day and for the last two days. Overall, the general pattern is not bad.

Figures 4.40 and 4.42 show that the QJM model mostly underestimates the German combat power and tank losses for the ACUD data set. This result is also supported by their signed rank test p-values that indicate that the difference between real and estimated losses is not zero. As in all other models, the battle is overestimated on the last days. This can be explained by the intensity of the battle which decreases after 12 July (8th day). Figure 4.41 presents the estimated versus real German personnel losses. The model fits the German personnel losses best among all other models. This result is apparent from its R^2 and signed rank test values.

Figure 4.46 shows the German combat power losses in the CCUD data. It has the same pattern as the ACUD data (Figure 4.40). Its signed rank test p-value shows that the model under/overestimates the battle. The same result is true for Figure 4.48, which

presents the German combat power losses in the FCUD data set. The Germans have lower combat power casualties when they are in a defensive position. These days are the 8^{th}, 9^{th}, 13^{th} and 14^{th} which can be seen from the figures.

Figure 4.43 shows the Soviet combat power losses. The model under/overestimates the battle. This is a similar pattern seen with the previous models (see Figure 4.5). The estimated Soviet combat power losses (Figure 4.43) and personnel losses (Figure 4.44) have different patterns. This suggests that combat power losses are highly affected by the tank losses.

Figure 4.42 presents the Soviet personnel losses. The signed rank test p-value shows that the difference between actual and estimated losses is zero. Compared with other weapon types, the Soviet personnel losses have a better fit than the others. This is the same result occurs with German personnel losses. This result suggests that the basic QJM equation, which predicts the personnel losses, fits the battle well. Further investigation is needed for the coefficients that are used in the computation of tank and artillery casualties based upon the personnel casualty rates.

When the different data sets are compared, there is no strong evidence that any of the models fits better than the others. However, overall, the ACUD data set fits slightly better than the other data sets for German losses.

Figure 4.45 shows the Soviet tank loss comparison. It is clear that the model overestimates the battle, which is also supported by the signed rank test. When the Soviets attacked, the model highly overestimates battle casualties compared with the other days. Even though the 8^{th} day was one of the heaviest days for tank battles, unlike

the previous models, the QJM model overestimates this day. One of the reasons is that the standard tank loss rate for the attacker is 12. Another reason might be the overwhelming German combat power ratio over the Soviets. The Soviet combat power ratio is very small, especially during the last days of the battle. For the Soviets, the ACUD data set fits better than the other two data sets. All these results should be considered when comparing QJM to the other models (ATLAS, RAND's SFS), as it is difficult to make direct comparison.

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V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

This thesis applies the Firepower score models to the data on the Battle of Kursk of World War II, which is considered the greatest single tank battle in history. Three Firepower score models were used. The first model is the ATLAS ground attrition model, which is used in the TACWAR simulation [Ref. 24]. The second model is RAND's SFS model, which was proposed in 1991 and is used in the JICM simulation [Ref. 25]. The last model is the simplified QJM model, developed by Trevor Dupuy.

This thesis is the first published study that compares the firepower score approach to calculate combat attrition with real two-sided, time-phased data. Three completely different firepower score models were applied to three different data sets. Instead of focusing only on one model and investigating it in detail, the applicability of the three primary firepower score models to real data was investigated. In addition, some insight is given about the attrition processes and other factors used in aggregated combat models.

Before presenting the conclusions of the study, we want to point out some of the assumptions and limitations already mentioned in different parts of the study. These assumptions and limitations follow.

- Although the data presented in the KOSAVE study is at the division level, all divisions were aggregated into one sector, which represents the southern front of the Battle of Kursk. The reason for this aggregation is the lack of data about which division was engaged with which opposing division. Thus, the data is highly aggregated.

- The KOSAVE II [Ref. 14] report uses nine weapon groups. In order to assign firepower score values to these weapon groups, RAND's proposed score values were used. Tanks were selected as the base weapon category and assigned a firepower score value of 100. Firepower score values are not assigned to each specific weapon type. Rather, it is assumed that all weapon types have the same score value in their weapon group. Therefore, all tank types have the same firepower score value of 100. Other weapon groups are assigned their firepower score values relative to RAND's score proportions. In a following study, firepower score values can be assigned to each specific weapon type rather than weapon groups. Furthermore, another approach to assigning them could be located.
- As mentioned in detail in Chapter IV Section A.2.e, a linear regression analysis was done to distribute the combat power casualty rate into different weapon groups. In the analysis, the real data on the Battle of Kursk was used. In advance of a battle, analysts would, of course, not know this.
- On the Eastern Fronts of the two World Wars, the German relative combat effectiveness superiority over the Russians was ranging between the factors of 2.0 and 3.0. [Ref. 3:p. 43]. The relative combat effectiveness value used in some models was chosen as 3 based on Turkes's research [Ref. 11]. Similar to the relative combat effectiveness factor, some of the factors used in different models are subjective and in follow on studies can be investigated in detail.
- Towards the end of the battle, the battle lost its intensity. Although most of the units on both sides were in a defensive position, in our models it is assumed that one side is attacking the whole sector. This caused an overestimation of the casualties in the models, especially, the models where battle intensity is not implemented explicitly.

One of the hardest and time-consuming parts of this thesis was to organize the data needed for the models used. Three different data sets were extracted from the data on the Battle of Kursk. These data sets are All Combat Unit Data (ACUD), Contact Combat Unit Data (CCUD) and Fighting Combat Unit Data (FCUD). This extraction is based upon the combat status of the units--active, contact, and fighting. The reason for this partitioning is to analyze the effects of the engagement status of the units over the models. One of the interesting aspects of this battle is the engagement percentages of the

forces. With an average of 97 percent of its heavy mechanized force on the front lines, the Germans, unlike the Soviets, had no reserves to use [Ref. 14:p.9-4].

The firepower score values are used to aggregate the different assets of the forces. The score values used in this thesis are computed relative to RAND's proposed firepower score values, which are presented in RAND's note, Situational Force Scoring (SFS) [Ref. 7]. Also, the values that Bracken used in his study [Ref. 8] are also used to compare the results with different weights. However, in his study, Bracken used only four groups of weapons. For the rest of the weapon groups, which are already defined in this study; RAND's values were again used. Thus, Bracken's weights were not completely used. This fact should be considered when comparing the models.

The firepower scores are assigned to weapon groups instead of specific weapon types. Thus, all the weapon types in a group have the same score. For instance, all tanks have the same firepower score regardless of their type. This aggregation causes the loss of information.

All three models are applied to the three different data sets. The result of each model is given in the section in which the model is described. The results obtained from the different models are intended to provide insight to the Battle of Kursk and aggregated combat modeling in general. In order to present the results and compare them to the other models, the R^2 values, given in Equation 4.6, are used. A greater R^2 value indicates a better fit. It is possible to get a negative R^2 value, implying that the fitted model yields worse results than using the average daily losses as an estimate.

Also, the estimated versus real losses are presented in the figures for each weapon group for both sides. These figures are very helpful in showing to show the pattern of the fits visually. In order to test the hypothesis of whether or not the difference between actual and estimated losses is the same, the Wilcoxon signed-rank test is used for all figures.

When all the models in all the data sets are viewed in general, the following conclusions are reached.

- Of all the models looked at, when combat power losses are considered, the ATLAS model with the air sortie data fits best.
- Generally, the models overestimate the attacker's casualties during the battle.
- Overall, all of the models fit better for the Germans than the Soviets. In his study [Ref. 11], Turkes also found that his models fit better for the Germans.
- In all of the models and for both sides, the FCUD data set gives the best fit.
- One of the difficulties with aggregated combat attrition models that use force ratio is the need to determine the attacking side. It is always not very easy to determine the attacking side. In this study, the battle is highly aggregated, and the data on the Southern Front of the Battle of Kursk is used. This area consists of more than 50 divisions. As described in Chapter III, the attacking side is determined by the percentage of the units that are attacking in a day. During the last days of the campaign, the attacking units percentages are very low for both sides. However, the attacking side is determined despite these low percentages. It is clearly seen that in almost all models, during the last days of the battle, the battle is overestimated.
- One way to overcome the problem described above might be to use the battle intensity factor in the models. The SFS and the QJM models implement this factor. However, one might argue the difficulty in predicting the battle's intensity beforehand.
- Prior to a battle, it is difficult to determine factors such as intensity and nationality factors.

- One of the problems with traditional force ratio models is that the loss rates in each weapons category are the same as the combat power casualty rate. For instance, if the combat casualty rate is 4 percent, then each weapons category will take 4 percent losses. However, this does not match either the historical facts or the results from the higher-resolution combat models. In this thesis, the linear regression analysis is used to determine how to allocate the combat power casualty rates to the different weapon groups.
- Including the air sorties in the models improved the quality of the fits for all models. The air sortie data includes the bombing and ground attack sorties for both the Germans and Soviets. The Germans generated more air sorties than the Soviets did during the campaign.
- Due to the general overestimation of the German casualties and the underestimation of the Soviet casualties, anything that improves the force ratio with respect to the Germans improves the quality of the fits. A better fit could probably be obtained by adding factors for German efficiency relative to the Soviets. Anything that added to German effectiveness or cut Soviet effectiveness could improve the quality of the fits.
- Partitioning the data according to the units' engagement status significantly improves the quality of the fits. When all the models are compared, the fits are better in the Fighting Combat Unit Data (FCUD) than the ACUD and CCUD data. The FCUD data represents the combat units directly engaged with the enemy.
- Using different firepower scores, like Bracken's weights, does not give a better fit except for the Soviet values in the CCUD data set, which is slightly better. Much more work is needed to find the best firepower scores, such as optimization of the score values, and sensitivity analysis.
- The ATLAS ground attrition algorithm is very straightforward. One of the concerns with the models is that the relative combat effectiveness of the forces is not implemented explicitly. Despite the fact that on the Eastern Fronts of the two World Wars the German relative combat effectiveness superiority over the Russians ranged between the factors of 2.0 and 3.0. [Ref. 3: p.43], in the ATLAS model they were assumed to be equal. This does not make sense since the force ratio of the Germans is lower than 1 in the ATLAS model. Thus, with this low force ratio, the Germans should not attack the overwhelming Soviet forces.
- The relative combat effectiveness is used in the SFS and the QJM models. When this factor is taken into account, the German force ratio over the Soviets increases, which gives a better explanation for their attacks.

- Although the SFS algorithm consists of a very simple calculation, it is hard to define the multipliers, which are used in the calculations. As mentioned in Chapter IV.B, some of the steps of the method are not clear.
- In the SFS model, the weapon categories are mapped into the three combined arms branches: armor, infantry, and artillery. At the end of the calculations, the combat power casualty rates are distributed into these three groups. This fact should be considered when comparing the SFS with other models, which distributed casualties into nine weapon groups.
- The best fit in the SFS model is observed in the German armor losses for the ACUD data set and the German combat power losses for the FCUD data set.
- Also in the SFS model, the FLOT movement rates are computed by the equations presented in RAND's notes. The best R^2 value for the estimated movement rates, 0.45, is computed in the ACUD data set, which is a very good fit for the combat models.
- The FLOT movement rate is only computed in the SFS model. The ATLAS model has also look up tables to compute the movement rates according to the force ratio, terrain, and combat postures. However, in the tables, the force ratio threshold is higher than the ones computed for this battle. As mentioned above, the force ratio of the Germans in the ATLAS model is very low. As a result, it was not possible to compute the FLOT movement rates in the ATLAS model.
- In the QJM model, the basic attrition calculation is for personnel losses. Losses for other weapon types and materials are based on their historical relationship to personnel losses. The R^2 value, 0.31, is the best fit for the German personnel losses in the ACUD data of all models. This result shows that the relationship between personnel losses and the other weapon groups should be examined further.
- Table 5.1 presents the R^2 values for all models for combat power and 10 weapon categories. The positive values are highlighted. For the SFS model, personnel represents the infantry, armor represents the tank and APC. Some models have negative R^2 values. This means one can have a better estimate of attrition just by using the mean value instead of using the model itself. In other words, it is better to use the mean value for estimating the attrition instead of using the estimate given by the models. Since the R^2 values are sensitive to a poor fit for one day, it is better to use them with comparison figures, which presents the pattern of the fits.
- In the SFS model, some R^2 values are negative and very low which indicates very poor fits. Likewise, in the QJM model, especially for the Soviet weapon groups, R^2 values are very low. The only positive values in

the QJM model are observed in the German personnel and combat power losses for the ACUD data set.

- In the ATLAS models, the APC R^2 values are mostly positive, which shows these models fit fairly well in the APC weapon group. The tank R^2 values are fairly good in all of the models for both forces. No positive value is seen for artillery, RKTL, MTR and AA weapon groups in all of the models for both sides.
- As mentioned in detail in Chapter III, some discrepancies are observed for the number of some weapons in the original database. These need to be investigated in the future.

Model	Data Set	Forces	Prsnl.	Tank	Arty.	APC	RKTL	ATH	MTR	ATL	Flame/ MG	AA	Combat Power
ATLAS Models Air Data is not Included	ACUD	German	-0.80	**0.16**	-0.63	-0.15	-0.11	-0.73	-0.43	-0.79	-0.41	-0.32	-0.01
		Soviet	-0.65	-0.19	-0.75	0.04	-0.91	-1.31	-1.72	-1.32	-1.69	-0.23	-0.71
	CCUD	German	-0.86	**0.26**	-0.70	-0.08	-0.13	-0.64	-0.59	-0.80	-0.42	-0.29	**0.08**
		Soviet	-0.33	-0.20	-0.45	**0.08**	-0.15	-0.78	-1.12	-0.92	-1.19	-0.26	-0.53
	FCUD	German	**0.16**	**0.54**	-0.30	**0.35**	-0.13	**0.05**	-0.08	-0.79	**0.00**	-0.25	**0.57**
		Soviet	-0.73	-0.21	-0.39	**0.40**	-0.22	-0.81	-1.67	-1.18	-1.67	-0.18	-0.76
ATLAS Models with Air Data	ACUD	German	-0.62	**0.24**	-0.42	-0.05	-0.08	-0.44	-0.39	-0.25	-0.30	-0.11	**0.11**
		Soviet	-0.24	**0.03**	-0.47	**0.14**	-0.42	-0.73	-0.82	-0.63	-0.75	-0.19	-0.23
	CCUD	German	-0.67	**0.33**	-0.46	**0.01**	-0.09	-0.39	-0.54	-0.25	-0.27	-0.09	**0.21**
		Soviet	-0.26	-0.15	-0.40	**0.10**	-0.13	-0.68	-1.01	-0.81	-1.07	-0.24	-0.44
	FCUD	German	**0.23**	**0.58**	-0.15	**0.40**	-0.10	**0.17**	-0.06	-0.28	**0.06**	-0.06	**0.62**
		Soviet	-0.40	-0.08	-0.26	**0.46**	-0.15	-0.51	-0.98	-0.70	-1.00	-0.14	-0.42
ATLAS Models With Air Data And Bracken's Weights	ACUD	German	-0.76	**0.18**	-0.66	-0.15	-0.13	-0.69	-0.42	-0.81	-0.39	-0.26	-0.28
		Soviet	-0.50	-0.08	-0.62	**0.09**	-0.82	-1.06	-1.39	-1.03	-1.32	-0.20	-0.57
	CCUD	German	-0.81	**0.28**	-0.71	-0.04	-0.14	-0.64	-0.55	-0.86	-0.38	-0.32	-0.24
		Soviet	-0.21	-0.12	-0.38	**0.10**	-0.12	-0.64	-0.92	-0.74	-0.99	-0.23	-0.34
	FCUD	German	**0.16**	**0.53**	-0.32	**0.36**	-0.14	**0.03**	-0.07	-0.85	**0.00**	-0.27	**0.45**
		Soviet	-0.63	-0.14	-0.34	**0.44**	-0.19	-0.70	-1.50	-1.04	-1.50	-0.16	-0.71
RAND's SFS Models (infantry, Armor, Artillery)	ACUD	German	-180.4	**0.59**	-131.5	-	-	-	-	-	-	-	-13.45
		Soviet	-48.15	-0.31	-69.36	-	-	-	-	-	-	-	-16.82
	CCUD	German	-32.47	**0.066**	-25.51	-	-	-	-	-	-	-	-0.51
		Soviet	-48.85	-0.52	-35.38	-	-	-	-	-	-	-	-17.76
	FCUD	German	-7.79	-0.28	-9.40	-	-	-	-		-	-	**0.38**
		Soviet	-43.73	-0.04	-28.09	-	-	-	-	-	-	-	-14.45
Dupuy's Simplified QJM Models	ACUD	German	**0.31**	-0.02	-1.74	-39.82	-	-1.81	-0.13	-1.31	-	-2.05	**0.20**
		Soviet	-27.72	-3.40	-119.5	-101.0	-	-336	-33.93	-198	-	-91	-2.49
	CCUD	German	-0.19	-0.29	-1.97	-23.52	-	-1.82	-0.13	-1.85	-	-2.17	-0.25
		Soviet	-15.80	-1.23	-87.98	-66.41	-	-120	-16.34	-100	-	-13	-1.07
	FCUD	German	-0.33	-0.48	-1.67	-11.54	-	-1.48	-0.01	-1.69	-	-1.96	-0.46
		Soviet	-14.61	-0.17	-87.48	-66.95	-	-83.99	-15.06	-76.55	-	-7.06	-0.76

Table 5.1. Results of All the Models Investigated in Chapter IV.

- This analysis is based on observational census data of the Battle of Kursk of World War II, and may not generalize, since it is not a random sample of a larger population. The outcome of a battle cannot be precisely determined with the use of combat models. They might provide insights into future battles between adversaries. Besides being used to gain insight into the battles, which occurred in the past, they should help in making better decisions by enabling the decision-maker to compare the different alternatives by using various combat model techniques [Ref. 11:p 145].

B. RECOMMENDATIONS

The data used in this study contains only combat units. The support units and the headquarters (HQ) above division level are not included. Data including the support units and HQs can be examined in future analyses.

In this study, the weapon groups presented in the KOSAVE study were used [Ref. 14]. They can be grouped differently by aggregating or disaggregating the weapon groups.

The firepower score values used for aggregating the forces are subject to much research. The best score values can be computed by an optimization that gives the best R^2 value. Also, a sensitivity analysis can be made on the best firepower score values for this battle. Score values were assigned to weapon groups. The database includes the necessary data to consider every weapon type. Assigning different firepower score values to each specific weapon type can be examined.

Partitioning the data according to the unit's engagement status can be applied to the other models, i.e., not firepower score models. For instance, Turkes's study [Ref. 11] can be revisited by using the three data sets presented in this thesis.

As mentioned in Chapter III, the Ardennes Campaign Simulation (ARCAS) Study [Ref. 18] contains a database of Ardennes Campaign World War II 1944-1945. All the models in this thesis can be applied to the Ardennes Campaign data set which is also two-sided, time-phased (daily) and highly detailed.

Although the data presented in the KOSAVE study is at the division level, all of the divisions are aggregated into a sector, which represents the southern front of the Battle of Kursk. The reason for this aggregation is the lack of data about which division was engaged with which opposing division. If someone can obtain this information from the battle maps or the source of the data, the whole sector can be broken down into multiple engagements. This higher resolution may improve the fits.

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